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## **APP-1065**

## M.A./M.Sc. (Previous) Examination, 2022 MATHEMATICS

Paper - I

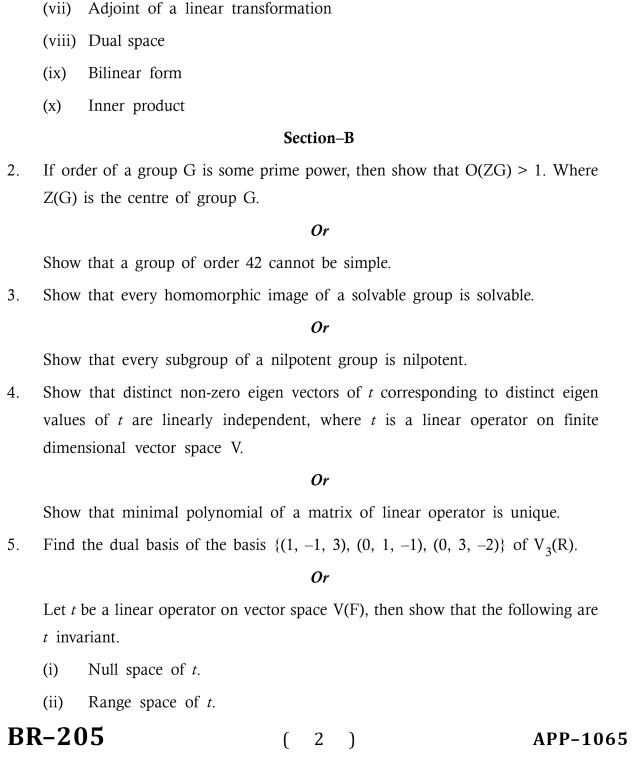
## (Advanced Abstract Algebra)

*Time* : **3** *Hours* ] [ Maximum Marks : 100 Section-A (Marks :  $2 \times 10 = 20$ ) Answer all ten questions (Answer limit 50 words). Each question carries Note :-2 marks. Section-B  $(Marks: 4 \times 5 = 20)$ Note :-Answer all five questions. Each question has internal choice (Answer limit 200 words). Each question carries 4 marks. Section-C (Marks :  $20 \times 3 = 60$ ) Answer any three questions out of five (Answer limit 500 words). Each *Note* :question carries 20 marks. Section-A

1. Define the following:

- (i) Commutator subgroup
- (ii) Algebraic extension
- (iii) Central ascending series

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Perfect field

Similar matrices

Minimal polynomial

(iv)

(v)

(vi)

6. Show that the function  $f: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$  defined as  $f(u, w) = u_1 w_2 - u_2 w_1$  is a bilinear form on  $\mathbb{R}^2 \times \mathbb{R}^2$ , where :

$$u = (u_1, u_2), w = (w_1, w_2)$$

$$Or$$

Show that  $K_n(C)$  is an inner product space for the inner product defined as:

$$(u, w) = u_1 \overline{w}_1 + u_2 \overline{w}_2 + \dots + u_n \overline{w}_n$$

where:

$$u = (u_1, u_2, ..., u_n), w = (w_1, w_2, ..., w_n)$$

## Section-C

- 7. (i) If L is an algebraic extension of K and if K is an algebraic extension of F, then show that L is an algebraic extension of F.
  - (ii) If f is a homomorphism of a group G into G', then show that :

$$\frac{G}{K_{\text{erf}}} \cong f(G)$$

- 8. (i) Show that any splitting field of a polynomial over field F is a normal extension of F.
  - (ii) Show that an infinite abelian group does not have a composition series.
- 9. Let  $f(x) = a_0 + a_1 x + \dots + a_n x^n$  be a polynomial with coefficients as integers. If p is a prime such that :

$$p \times a_{n}, p/a_{n-1}, p/a_{n-2}, ...., p/a_{0} \text{ and } p^{2} \times a_{0}$$

then show that f(x) is irreducible over the field of rational numbers.

- 10. (i) Show that a linear transformation P on vector space V(F) is a projection on some subspace if it is idempotent.
  - (ii) Show that:

$$(t_1 + t_2)^* = t_1^* + t_2^*$$

where  $t^*$  is the adjoint of linear transformation t.

)

- 11. (i) State and prove Cauchy's Schwarz inequality.
  - (ii) If F is a field of characteristic ( $\neq$  2), then show that every symmetric bilinear form on vector space V(F) is uniquely determined by the corresponding quadratic form.