Roll No. : ....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

APP–1065

M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper - I

(Advanced Abstract Algebra)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. Define the following :

   (i) Commutator subgroup

   (ii) Algebraic extension

   (iii) Central ascending series

(iv) Perfect field

(v) Similar matrices

(vi) Minimal polynomial

(vii) Adjoint of a linear transformation

(viii) Dual space

(ix) Bilinear form

(x) Inner product

Section–B

2. If order of a group G is some prime power, then show that $O(ZG) > 1$. Where $Z(G)$ is the centre of group G.

Or

Show that a group of order 42 cannot be simple.

3. Show that every homomorphic image of a solvable group is solvable.

Or

Show that every subgroup of a nilpotent group is nilpotent.

4. Show that distinct non-zero eigen vectors of t corresponding to distinct eigen values of t are linearly independent, where t is a linear operator on finite dimensional vector space V.

Or

Show that minimal polynomial of a matrix of linear operator is unique.

5. Find the dual basis of the basis $\{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ of $V_3(R)$.

Or

Let t be a linear operator on vector space V(F), then show that the following are t invariant.

(i) Null space of t.

(ii) Range space of t.

6. Show that the function $f : R^2 \times R^2 \to R$ defined as $f(u, w) = u_1 w_2 - u_2 w_1$ is a bilinear form on $R^2 \times R^2$, where :

$$u = (u_1, u_2), \; w = (w_1, w_2)$$

Or

Show that $K_n(C)$ is an inner product space for the inner product defined as :

$$(u, w) = u_1\overline{w}_1 + u_2\overline{w}_2 + \ldots\ldots + u_n\overline{w}_n$$

where :

$$u = (u_1, u_2, \ldots., u_n), \; w = (w_1, w_2, \ldots, w_n)$$

Section–C

7. (i) If L is an algebraic extension of K and if K is an algebraic extension of F, then show that L is an algebraic extension of F.

   (ii) If f is a homomorphism of a group G into G′, then show that :

$$\frac{G}{K_{erf}} \cong f(G)$$

8. (i) Show that any splitting field of a polynomial over field F is a normal extension of F.

   (ii) Show that an infinite abelian group does not have a composition series.

9. Let $f(x) = a_0 + a_1 x + \ldots. + a_n x^n$ be a polynomial with coefficients as integers. If p is a prime such that :

$$p \times a_n, \; p/a_{n-1}, \; p/a_{n-2}, \; \ldots., \; p/a_0 \text{ and } p^2 \times a_0$$

then show that $f(x)$ is irreducible over the field of rational numbers.

10. (i) Show that a linear transformation P on vector space V(F) is a projection on some subspace if it is idempotent.

   (ii) Show that :

$$(t_1 + t_2)^* = t_1^* + t_2^*$$

   where t^* is the adjoint of linear transformation t.

11. (i) State and prove Cauchy's Schwarz inequality.

(ii) If F is a field of characteristic ($\neq 2$), then show that every symmetric bilinear form on vector space V(F) is uniquely determined by the corresponding quadratic form.