Roll No. : ....

Total No. of Questions : **16**] [Total No. of Printed Pages : **4**

MATHSEM–119

M.A./M.Sc. (Ist Semester) Examination Dec., 2022

MATHEMATICS

Paper – III

(Tensor Analysis)

*Time : **3** Hours*] [*Maximum Marks : **50***

The question paper contains three Sections.

Section–A **(Marks : 1 × 9 = 9)**

Note :– Answer all the *nine* questions carries **1** mark each. The answer should not exceed **50** words.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer *five* questions by selecting at least *one* question from each Unit. Each question carries **4** marks. Answer should not exceed **200** words.

Section–C **(Marks : 7 × 3 = 21)**

Note :– Answer *three* questions by selecting at least *one* question from each Unit. Each question carries **7** marks. The answer should not exceed **500** words.

Section–A

1. (i) Define conjugate or reciprocal symmetric tensor.

   (ii) If A^{ij} is Skew-symmetric and B_{ij} is symmetric, prove that $A^{ij}B_{ij} = 0$.

   (iii) Find the condition of orthoganality of two vectors A^i and B^i.

   (iv) Evaluate :

$$[i\,j, k] + [k\,j, i]$$

   (v) Obtain the necessary and sufficient condition for a vector B^i, if variable magnitude to suffer a parallel displacement along a curve C is that $B^i_{ij}\dfrac{dx^j}{ds}$.

   (vi) Define Geodesics.

   (vii) What is the condition for flat space ?

   (viii) Find the value of :

$$R^{\beta}_{ijk} + R^{\beta}_{jki} + R^{\beta}_{kij}$$

   (ix) Define Covariant curvature tensor.

Section–B

Unit–I

2. Prove that Kronecker delta is a mixed tensor of rank two and it is invariant.

3. Prove that outer multiplication of tensor is commutative and associative.

4. Prove that (1, 0, 0, 0) and $\left(\sqrt{2}, 0, 0, \sqrt{3}/c\right)$ are unit vectors in V_4 with the metric :

$$ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$$

   Also prove that the angle between these vectors is not real.

Unit–II

5. If g_{ij} and a_{ij} are components of two symmetric covariant tensors and $\begin{Bmatrix} i \\ jk \end{Bmatrix}_g$, $\begin{Bmatrix} i \\ jk \end{Bmatrix}_a$ are the corresponding Christoffel symbols of the second kind then prove that the quantities :

$$\begin{Bmatrix} i \\ jk \end{Bmatrix}_g - \begin{Bmatrix} i \\ jk \end{Bmatrix}_a$$

are components of a mixed tensor.

6. Show that the pole P_0 of the geodesic coordinate system :

$$A_{i,jk} = \frac{\partial^2 A_i}{\partial x^j \partial x^k} - Al \frac{\partial}{\partial x^k}\begin{pmatrix} l \\ ij \end{pmatrix}$$

7. Show that the covariant derivative of an invariant is the same as its ordinary derivative.

Unit–III

8. For a V_2 referred to an orthogonal system of parametric curve, show that :
   (i) $R_{12} = 0$
   (ii) $R_{11}g_{22} = R_{22}g_{11} = R_{1221}$
9. Define Ricci Tensor and explain its properties.
10. Show that a geodesic is an auto parallel curve.

Section–C

Unit–I

11. Prove that the fundamental tensor g_{ij} is a covariant symmetric tensor of rank two.
12. Prove that :
    (i) $g^{ij}g^{kl}dg_{ik} = -dg^{jl}$
    (ii) $g_{ij}g_{kl}dg^{ik} = -dg_{jl}$

Unit–II

13. The covariant derivative of a covariant vector is symmetric if and only if the vector is gradient.

14. Show that the covariant derivatives of the tensors g_{ij}, g^{ij} and δ all vanish identically.

Unit–III

15. State and prove Bianchi identity.

16. If the metric of a two dimensional flat space is :

$$ds^2 = f(r)[(dx^1)^2 + (dx^2)^2],$$

show that $f(r) = c(r)^k$, where $r^2 = (x^1)^2 + (x^2)^2$ and c, k are constants.