Roll No. : ....

Total No. of Questions : **16**] [Total No. of Printed Pages : **3**

MATHSEM–117

M.A./M.Sc. (Ist Semester) Examination Dec., 2022

MATHEMATICS

Paper – I

(Advanced Abstract Algebra)

*Time : **3** Hours*] [*Maximum Marks : **50***

The question paper contains three Sections.

Section–A **(Marks : 1 × 9 = 9)**

Note :– The candidate is required to answer all the *nine* questions carries **1** mark each. The answer should not exceed **50** words.

Section–B **(Marks : 4 × 5 = 20)**

Note :– The candidate is required to answer *five* questions by selecting at least *one* question from each Unit. Each question carries **4** marks. Answer should not exceed **200** words.

Section–C **(Marks : 7 × 3 = 21)**

Note :– The candidate is required to answer *three* questions by selecting *one* question from each Unit. Each question carries **7** marks. The answer should not exceed **500** words.

Section–A

1. Define the following :

   (i) p-Sylow subgroup.

   (ii) kth centre of a group.

   (iii) Length of subnormal series.

   (iv) Primitive polynomial.

   (v) Euclidean ring.

   (vi) Noetherian ring.

   (vii) Right module.

   (viii) Cyclic module.

   (ix) Simple module.

Section–B

Unit–I

2. Show that :

$$\frac{G}{G} \cong \{e\}$$

3. If the kth derived group of a group G is the identity group, then show that G is solvable.

4. Show that a finite p-group has a non-trinial centre.

Unit–II

5. Show that the polynomial $(x^2 + 1)$ is irreducible over the field (z_7, t_7, x_7).

6. Show that the ring of Gaussian integers is a Euclidean ring.

7. Show that the ring of integers is a Noetherian ring.

Unit–III

8. Show that every abelian group G is module over the ring of integers.

9. Let A be a Noetherian ring and M be a finitely generated module. Then show that M is Noetherian.

10. If $f : M \rightarrow M'$ be an isomorphism of modules, then show that Kerf $= \{0\}$.

Section–C

Unit–I

11. If a prime number p divides the order of a finite group G, then show that there exists an element C ($\neq e$) in G such that O(c) = P.

12. Let f be a homomorphism of a group G onto a group G′. Let N′ be normal subgroup G′ and :

$$f^{-1}(N') = \{\alpha \in G \text{ s.t. } f(\alpha) \in N'\}$$

then show that :

$$\frac{G}{f^{-1}(N')} \cong \frac{G'}{N'}$$

Unit–II

13. Show that every non-zero element of a Euclidean ring R is either a unit of R or can be written as a product of a finite number of prime elements of R.

14. If $f(x)$ and $g(x) \neq 0$ are any two polynomials over a field F, then show that there exists unique polynomials $q(x)$ and $r(x)$ in F(x) such that :

$$f(x) = q(x)\, g(x) + r(x)$$

where either $r(x) = 0$ or deg $r(x) <$ deg $g(x)$.

Unit–III

15. Let R be any ring and I be a left ideal of R, then show that :

$$A = \{\alpha + I, \alpha \in R]$$

is an R-module.

16. Let R be a Euclidean ring, then show that any finitely generated R-module M is the direct sum of a finite number of cyclic modules.