Roll No. : .....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

APF–2169

M.A./M.Sc. (Final) Examination, 2022

MATHEMATICS

Paper - Opt-II

(Advanced Discrete Mathematics)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) Define Tautologies and Contradictions.

   (ii) Define Semi-group with example.

   (iii) Define Lattices.

   (iv) Define Distributive Lattices.

(v) Write the statement of Kuratowski's theorem.

(vi) Define Planar Graph.

(vii) Define Spanning Tree.

(viii) Define Rooted Tree.

(ix) Define Finite State Automata.

(x) Define Phrase-structure Grammar.

Section–B

2. If $f : G \to G'$ be a homomorphism then :

(i) $f(e) = e'$ where e, e' are identities of G & G′

(ii) $f(a^{-1}) = (f(a))^{-1}$.

Or

Prepare truth table for the statement : $[(p \to q) \wedge (q \to p)]$.

3. If $(L, \leq)$ is a lattice with binary operations $\vee$ and $\wedge$, then for arbitary element a, b, c, $d \in L$:

(i) $a \leq b$ and $c \leq d \Rightarrow a \wedge c \leq b \wedge d$

(ii) $a \leq b$ and $c \leq d \Rightarrow a \vee c \leq b \vee d$

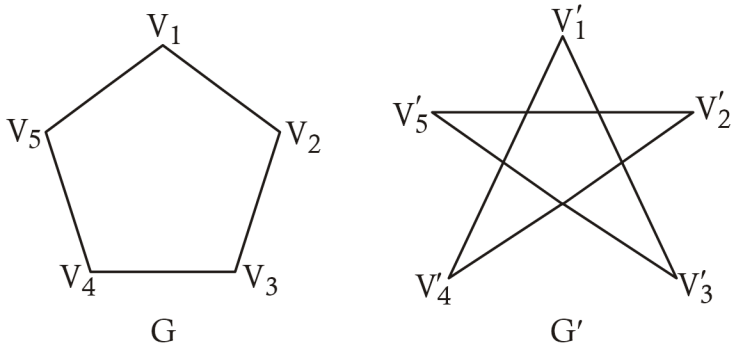
Or

Let $< B, +, \cdot, ', 0, 1 >$ be a Boolean algebra, then for all elements $a \in B$:

(i) $a + 1 = 1$

(ii) $a \cdot 0 = 0$

4. The graph G and G′ shown in the following figure are isomorphic.



Or

Explain Dijkstra algorithm with example.

5. Every tree has either one or two centres.

Or

Explain Kruskal's algorithm.

6. Explain Moore and Mealy machines.

Or

Show that the language $L = \left\{ a^i b^i : i \geq 1 \right\}$ is not regular.

Section–C

7. Prove that :

   (i) Every cyclic monoid is a commulative monoid.

   (ii) The identity element in any monoid is unique.

   (iii) Prove that the intersection of two submonoids of a monoid (M, *) is again a submonoid of (M, *)

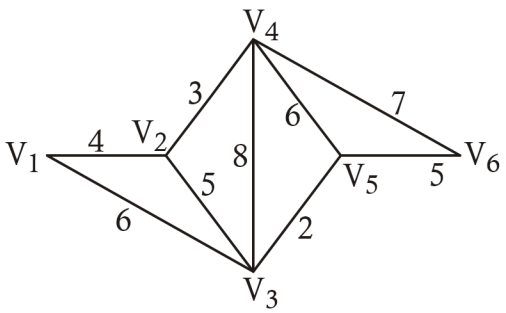
8. (i) Define Lattice and also prove that the dual of a lattice is also a lattice.

   (ii) Let < B, +, ·, ′, 0, 1 > be a Boolean algebra, then for all elements $a \in B$:

   (a) $a + a = a$

   (b) $a \cdot a = a$

9. (i) Define weighted graph and find the shortest path from the vertex V_1 to vertex V_6 in the following graph :



(ii) Draw graph, which are :

(a) Neither Euler nor Hamiltonian

(b) Euler but not Hamiltonian

(c) Hamiltonian but not Euler

(d) Both Euler and Hamiltonian

10. (i) A graph G is connected if and only if it has a spanning tree.

(ii) Define Prim's algorithm with example.

11. (i) State and proof Kleen's theorem.

(ii) Show that $L = \left\{ a^{2n} : n \geq 1 \right\}$ is a regular language.