Roll No. : ....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

APP–1069

M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper – V

(Numerical Methods)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A 2 each

1. (i) Write down Newton's formula for finding out square root of a number.

 (ii) Find out the initial approximate root of the equation $x \log_{10} x = 4.7772393$.

(iii) Divide $x^5 - 2x^4 + 3x^2 + 4x - 1$ by $(x - 3)$, using synthetic division and obtain quotient and remainder (one step).

(iv) Define Relaxation Method.

(v) Write down the principle of method of least square for fitting a curve.

(vi) State Orthogonal Property of Chebyshev Polynomial.

(vii) Write down the formula of Euler's modified method for ordinary linear differential equation of first order.

(viii) Explain briefly the Multi-step Methods.

(ix) What is the difference between Initial Value Problem (IVP) and Boundary Value Problem (BVP) ?

(x) Write the names of *two* methods by which boundary value problem can be solved.

Section–B 4 each

2. Solve $(12)^{1/2}$ by applying Newton's formula up to three places of decimal.

Or

Find the real root of the equation $x^2 + 4 \sin x = 0$ correct to four places of decimals by using Newton Raphson method.

3. Solve the following system of equations by Gauss-Seidal iteration method :

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

$$x + y + 54z = 110$$

Or

Perform two iterations of the Birge-Vieta method to find the smallest positive root of the polynomial $p_3(x) = 2x^3 - 5x + 1 = 6$ use the initial approximation $p_0 = 0.5$.

4. Fit a straight line to the following data :

x	1	2	3	4	5
y	2	5	3	8	7

Or

Use power method to find the largest eigenvalue and corresponding eigenvector of the matrix :

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

with initial vector $Y_0 = [1 \quad 0 \quad 0]^T$.

5. Given :

$$\frac{dy}{dx} = \frac{y^2 - x}{y^2 + x}, \; x = 0, \; y = 1.$$

Compute y (0.1), y (0.2) by Euler's method.

Or

Use Taylor's series method to find the numerical solution of the :

$$\frac{dy}{dx} = x^2 + y^2$$

with $x = 1$, $y = 0$ at $x = 1.2$ (two steps).

6. Use Finite Difference method to solve the BVP :

$$\frac{d^2y}{dx^2} = y; \; y(0) = 0, \; y(1) = 1.8 \text{ with } h = \frac{1}{4}.$$

Or

Explain shooting method for Boundary Value Problem.

Section–C

7. (i) Find double root of the equation :

$$x^4 - 6.75x^3 + 6.25x - 1.5 = 0,$$

starting with $x_0 = 0.3$ using Newton Raphson Method.

(ii) Find a real root of the equation :

$$x^3 - 2x - 5 = 0$$

using Secant method, with two initial approximations $x_{-1} = 2$, $x_0 = 3$. 10+10

8. Find the roots of the equation :

$$x^3 - 2x - 5 = 0$$

by Birge-Vieta method correct up to two places of decimals. 20

9. Using the Chebyshev polynomials, obtain the least square approximation of second degree for $f(x) = x^4$ on $[-1, 1]$. 20

10. Solve the equation :

$$\frac{dy}{dx} = x + y$$

with initial condition $y(0) = 1$ by Runge-Kutta rule, from $x = 0$ to $x = 0.2$ with $h = 0.1$. 20

11. Solve the Boundary Value Problem :

$$y''(x) = y(x); \; y(0) = 0, \; y(1) = 1.1752$$

by the shooting method, taking $m_0 = 0.8$ and $m_1 = 0.9$. 20