Roll No. : ....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

APP–1068

M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper - IV

(Differential and Integral Equations)

Time : 3 Hours] [*Maximum Marks : 100*

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) Solve $\frac{dy}{dx} = \sqrt{|y|}$, $y(0) = 0$

(ii) Classify the following PDE :

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x \partial y}+\frac{\partial^2 u}{\partial y^2}=0$$

(iii) Explain Eigen function expansion.

(iv) Define Green's function.

(v) Write the necessary condition for $I=\int_{x_1}^{x_2} f(x, y, y')\,dx$ to be an extremum.

(vi) Define Functionals.

(vii) Define linear integral equations.

(viii) Define homogeneous Fredholm integral equation.

(ix) Explain the solution of Volterra integral equations by the method of successive approximations.

(x) Find the resolvent kernels if $k(x,t)=2x$, $\lambda=1$.

Section–B

2. Show that the solution $y(x)$ of IVP :

$$\frac{dy}{dx}=y^2+\cos x^2, \quad y(0)=0$$

exist for $|x| \le \frac{1}{2}$.

Or

Obtain the GS of wave equation $\frac{\partial^2 u}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$.

3. Prove that all the eigenvalues of the Sturm-Liouville problem are real.

Or

Use Green's functions technique to obtain solution of :

$$\frac{d^2 y}{dx^2}+k^2 y(x)=f(x), \quad 0 \le x \le L$$

Subject to the BC : $y(0)=0$, $y(L)=0$.

4. Find the shortest curves joining two points (x_1, y_1) and (x_2, y_2).

Or

Find a function $y(x)$ for which $\int_0^1 \left[x^2 - (y')^2\right] dx$ is stationary, given that :

$$\int_0^1 y^2 dx = 2\,,\ \ y(0) = 0\,,\ \ y(1) = 0\,.$$

5. Show that the function $g(x) = 1 - x$ is a solution of the integral equation :

$$\int_0^x e^{x-t} g(t)\, dt = x\ .$$

Or

Solve $g(x) = \cos x + \lambda \int_0^{\pi} \sin x\ g(t)\, dt$.

6. Find the resolvent kernels for Volterra integral equations with the following kernels :

$$k(x,t) = x - t$$

Or

Solve by classical Fredholm theory :

$$g(x) = e^x + \lambda \int_0^1 2e^x e^t\ g(t)\, dt$$

Section–C

7. (i) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\, \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

(ii) Find general solution of one-dimensional heat equation in Cartesian form :

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$

8. (i) Determine the normalized eigenfunctions of problem :

$$y'' + \lambda y = 0$$
$$y(0) = 0$$
$$y'(1) + y(1) = 0$$

(ii) Solve the BVP $-y'' = f(x)$, $y(0) = 0$, $y(1) + y'(1) = 0$ by determining the Green's function and expressing the solution as a definite integral.

9. (i) If a particle moving with a force perpendicular to and proportional to its distance from the line of zero velocity, show that the path of quickest descent is a circle.

(ii) Find the equations of the curves for which the functional

$$\int_0^1 \left[(y')^2 + 12xy \right] dx$$

can be extremised. Also given that $y(0) = 0$ and $y(1) = 1$.

10. (i) Form an integral equation corresponding to the differential equation

$$\frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = 0$$

with the initial conditions $y(0) = 1$; $y'(0) = 0$.

(ii) Using iterative method, solve the following integral equation

$$g(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xt \; g(t) \; dt$$

11. (i) Solve the following integral equation :

$$g(x) = 1 + \lambda \int_0^{\pi} \sin(x+t) \; g(t) \; dt$$

(ii) Solve the following integral equation, with the aid of resolvent kernel :

$$g(x) = 1 + \lambda \int_0^{x} e^{3(x-t)} \; g(t) \; dt$$