Roll No. : ....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

APP–1067

M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper - III

(Mathematical Methods)

Time : 3 Hours] [*Maximum Marks : 100*

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) Write the statement of Gauss theorem for hypergeometric function.

   (ii) Write the orthogonality property formula for Legendre polynomials.

   (iii) Write the integral representation of Bessel function.

(iv) Expand x^n in series of Hermite polynomial.

(v) Find the Laplace transform of :

$$L\{\sin^2 3x\}$$

(vi) Find the inverse Laplace transform of :

$$L^{-1}\left\{\frac{1}{p}\sin\frac{1}{p}\right\}$$

(vii) Define inner product of tensors.

(viii) Define Permutation tensor.

(ix) Define Christoffel symbol of first kind and second kind.

(x) Prove that :

$$\int_0^{\infty} e^{-pt} L_n(t)\, dt = \frac{1}{p}\left(1-\frac{1}{p}\right)^n$$

Section–B

2. State and prove Kummer's first transformation theorem.

Or

Prove the result :

$$_2F_1(-n, \alpha + n; \gamma; 1) = \frac{(-1)^n (1+\alpha-\gamma)_n}{(\gamma)_n}$$

3. Prove that :

$$(n + 1) P_n(x) = P'_{n+1}(x) - xP'_n(x)$$

Or

Prove that :

$$2J'_n(x) = J_{n-1}(x) - J_{n+1}(x)$$

4. Prove that :

$$H''_n(x) = 2x H'_n(x) - 2n H_n(x)$$

Or

If :

$$A_{ij} = 0 \quad i \neq j$$
$$A_{ij} \neq 0 \quad i = j$$

then prove that :

$$B^{ij} = 0 \qquad \text{if} \quad i \neq j$$
$$B^{ii} = \frac{1}{A_{ii}} \quad \text{if} \quad i = j$$

5. Prove that g_{ij} is a covarient tensor of rank 2.

Or

Prove the result :

$$[ij, h] = g_{kh} \begin{bmatrix} & k \\ i, & j \end{bmatrix}$$

6. Find the Laplace transform of $L\{xe^{ax} \sin bx\}$.

Or

Solve the Laplace differential equation $(D^2 + 1)\, y = 0$ where :

$$y(0) = 1$$

$$y'(0) = 0$$

Section–C

7. Show that :

$$\int_x^{\infty} e^{-y} L_n^{\alpha}(y)dy = e^{-x}\left[L_n^{\alpha}(x) - L_{n-1}^{\alpha}(x)\right]$$

8. Prove that :

$$e^{-x^2} H_n(x) = (-1)^n \frac{d^n}{dx^n}\left(e^{-x^2}\right)$$

9. Prove that the law of transformation of covarient tensor form a group or possess transitive property.

10. State and prove Ricci's theorem for covarient derivatives of tensors.

11. Prove for Laplace transform of exponential function :

$$L\left\{E_i(x)\right\} = \frac{1}{p}\log(p+1)$$