Roll No. : ....

Total No. of Questions : **11**]   [Total No. of Printed Pages : **4**

APP–1066

M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper - II

(Analysis)

Time : 3 Hours]   [*Maximum Marks : 100*

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) Define measurable function.

(ii) Define Countable set.

(iii) Define summable functions.

(iv) Define the indefinite Lebesgue integral.

(v) State Cauchy's theorem.

(vi) Find Laurent's series of the function $f(z) = \frac{1}{z^2(1-z)}$ about $z = 0$

(vii) Write down the names of three types of singularity.

(viii) State Roche's theorem.

(ix) State Cauchy's Residue theorem.

(x) Define Analytic continuation.

Section–B

2. Prove that a subset of countable set is countable.

Or

Prove that ϕ and R are measurable sets.

3. Prove that space L_2 of square summable functions is a linear space.

Or

If f be a bounded measurable function on a measurable set E, then show that

$$\left|\int_E f(x)\,dx\right| \le \int |f(x)|\,dx .$$

4. If $f(z)$ is analytic within and on a closed contour C, and if α is any point within C then show that :

$$f(\alpha) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-\alpha}\,dz$$

Or

Expand $f(z)=\dfrac{(z-2)(z+2)}{(z+1)(z+4)}$ in a Laurent's series valid for $1 < |z| < 4$.

5. If $z = \alpha$ is an isolated singularity of $f(z)$ and if $f(z)$ is bounded on some deleted neighbourhood of α, then show that α is a removable singularity.

Or

Find kind of singularities of $f(z)=\dfrac{\cot \pi z}{(z-\alpha)^2}$ at $z = \alpha$ and $z = \infty$.

6. Evaluate the residues of $f(z)=\dfrac{e^z}{z^2(z^2+9)}$ at $z = 0, 3i, -3i$.

Or

Evaluate by the method of calculus of residue $\int_c \dfrac{dz}{(z-1)(z+1)}$ where c is circle $|z| = 3$.

Section–C

7. State and prove Weierstrass' approximation theorem of continuous function by polynomials. 20

8. (i) If f is a measurable function of a measurable set E and if $a \le f(x) \le b$, then

$$a.m(\text{E}) \le \int_E f(x)dx \le b.m(\text{E})$$

(ii) If f and g are summable functions on a set E, then prove that $f \pm g$ is also summable and $\int_E [f(x) \pm g(x)]dx = \int_E f(x)dx \pm \int_E g(x)dx$ 10+10=20

9. (i) Define analyatic function.

(ii) Write down the Cauchy-Riemann equations for analytic function.

(iii) If $f(z) = \begin{cases} \dfrac{x^3 y(y - ix)}{x^6 + y^2} & , \quad z \neq 0 \\ 0 & , \quad z = 0 \end{cases}$ then show that $f(z)$ is not analytic at $z = 0$. 3+2+15=20

10. (i) Define singular points.

(ii) Define Branch points.

(iii) State and prove Casorati-Weierstrass theorem. 2+2+16=20

11. By method of contour integration, show that :

$$\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} = \frac{\pi}{6}$$

20