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Total No. of Questions : 11 ]

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# **APF-2174**

**M.A./M.Sc. (Final) Examination, 2022**

**MATHEMATICS**

Paper - Opt. VII

**(Mathematical Theory of Statistics)**

*Time : 3 Hours ]*

*[ Maximum Marks : 100*

**Section-A**

**(Marks : 2 × 10 = 20)**

*Note :-* Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

**Section-B**

**(Marks : 4 × 5 = 20)**

*Note :-* Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

**Section-C**

**(Marks : 20 × 3 = 60)**

*Note :-* Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

**Section-A**

1. (i) State Chebyshev inequality.
- (ii) What is the mathematical expectation of the sum of points on  $n$  dices?
- (iii) Write down regression line  $y$  on  $x$  and  $x$  on  $y$ .

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- (iv) Write down formula for Karl Pearson coefficient of correlation.
- (v) Define Z distribution.
- (vi) Write M.G.F. of  $\chi^2$ -distribution.
- (vii) Define tests of significance for difference of means.
- (viii) Write the formula for coefficient of association between two attributes.
- (ix) What do you mean by efficient and sufficient estimator ?
- (x) Describe Fisher's criteria for the best estimation.

**Section-B**

2. State and prove Bay's theorem.

*Or*

A fair die is tossed 720 times use Chebyshev's inequality to find a lower bound for the probability of getting 100 to 140 sixes.

3. Calculate the coefficient of correlation between  $x$  and  $y$  using the following data :

$x$	1	2	3	4	5	6	7	8	9
$y$	9	8	10	12	11	13	14	16	15

*Or*

In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the parameter's of the distribution. [ $\phi(1.4) = 0.42$ ,  $\phi(0.5) = 0.19$ ]

4. The following mistakes per page were observed in a book :

<b>No. of heads</b>	0	1	2	3	4
<b>Frequency</b>	211	90	19	5	0

Fit a Poisson distribution and test the goodness of fit.

*Or*

Given below are the qualities of 10 items (in proper units) produced by two processes A and B :

<b>Process A</b>	3	7	5	6	5	4	4	5	3	3
<b>Process B</b>	8	5	7	8	3	2	7	6	5	7

Test whether the variable of quality may be taken to be the same for the two processes.

[Given F-value for  $n_1 = 9$  and  $n_2 = 9$  degree of freedom is 3.18 at 5% level of significance and 5.35 at 1% level of significance.]

5. Calculate the coefficients of association between intelligence in father and son from the following data :

Intelligent father with intelligent sons = 248

Intelligent father with dull sons = 81

Dull father with intelligent sons = 29

Dull father with dull sons = 579

*Or*

The mean life time of a sample of 25 bulbs is found as 1550 hrs with standard deviation of 120 hrs. The manufacturing company claims that the average life of their bulbs is 1600 hrs. Is the claim acceptable at 5% level of significance.

6. Find the maximum likelihood estimate for the parameter  $\lambda$  of a Poisson distribution on the basic of a sample of size  $n$ . Also find its variance.

*Or*

If  $x_1, x_2, \dots, x_n$  be random sample of size  $n$  from the normal distribution with mean  $\bar{x}$  and variance  $\sigma^2$ , show that the statistic :

$$s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

is not an unbiased estimator of the parameter  $\sigma^2$ .

### Section-C

7. Thirteen cards are drawn from a deck of 52 cards if ace count 1, face cards 10 and other according to denominations, find the expected total.
8. If  $\theta$  is the acute angle between two regression lines in case of two variable  $x$  and  $y$ ; show that :

$$\tan \theta = \frac{1-r^2}{\sigma} \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

where  $r$ ,  $\sigma_x$ ,  $\sigma_y$  have their usual meanings. Explain the significance of the formula when  $r = 0$  and  $r = \pm 1$ .

9. For a chi-square distribution with *n.d.f.* establish the following recurrence relation between the moments :

$$\mu_{r+1} = 2r(\mu_r + n\mu_{r-1}); r \geq 1$$

hence find  $\beta_1$  and  $\beta_2$ .

10. Two sales persons A and B are working in a certain district and the following result were obtained from a sample. State whether there is any significant difference in the average sales between the two sales persons :

	<b>A</b>	<b>B</b>
No. of sales	20	18
Average sales (in ₹)	170	205
Standard deviation	20	25

Assume that there is no significant difference in the average sales of the two sales persons.

11. Given the frequency function :

$$f(x, \theta) = \frac{e^{-x^2/2\theta^2}}{\theta\sqrt{2\pi}}; \quad -\infty < x < \infty$$

find the maximum likelihood estimator for  $\theta$ .