Roll No. : ....

Total No. of Questions : **11**] [Total No. of Printed Pages : **3**

APF–2173

M.A./M.Sc. (Final) Examination, 2022

MATHEMATICS

Paper - Opt. VI

(Topology)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. Define the following :

   (i) Topological Spaces.

(ii) Homeomorphism Bases.

(iii) Complete Metric Spaces.

(iv) Locally Compact Spaces.

(v) Compactness.

(vi) Hausdorff Spaces.

(vii) Closed Subgroups.

(viii) Topology on the space of right.

(ix) Compact Group.

(x) Right Haar Measure.

Section–B

2. Write All Topologies for the set X = {1, 2, 3}.

Or

Show that the union of an Infinite Collection of closed sets in a topological space is not necessarily closed.

3. Let X = {1, 2, 3} and Let F = {[1, 2], X} be collection of subsets of X, then show that F is filter on X.

Or

Show that any open subspace of a locally compact space is locally compact.

4. Give *two* examples of locally connected spaces which are not connected.

Or

Show that every subspace of a Hausdorff-space is Hausdorff.

5. Let X = {a, b, c} and T = {ϕ, [a], [b, c], [X]}, then show that (X, T) is regular space but not T_2,

Or

Define Topological group. Prove that a T_0 topological group is a Tychonoff space.

6. Write short note for compact group.

Or

Show that every locally compact group admits a Left Haar Measure.

Section–C

7. Let (X, T) and (Y, V) be two topological spaces and let f be a one-one onto mapping of X on to Y. Then the following statements are equivalent :

   (i) f is open and continuous

   (ii) f is a Homomorphism

   (iii) f is closed and continuous

8. Prove that a topological space X is normal iff for any closed set F and open Set G containing F, then exists an open set G* such that : $F \subseteq G^*$ and $\bar{G}^* \subseteq G$.

9. If $f : X \to Y$ is continuous and Y is Hausdorff, then $A = \{(x, y) : f(x) = f(y)\}$ is a closed subset of XxX.

10. Prove that regular space is a topological property.

11. Explain existence and uniqueness of Left Haar Measure.