Roll No. : ....

Total No. of Questions : **11**]   [Total No. of Printed Pages : **7**

APF–2172

M.A./M.Sc. (Final) Examination, 2022

MATHEMATICS

Paper – Opt. V

(Operations Research)

Time : 3 Hours]   [*Maximum Marks : 100*

Section–A   **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B   **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C   **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) What is a basic Feasible solution of a Linear Programming Problem (LPP) ?

   (ii) When and why artificial variables are used in solving LPP by simplex method?

(iii) What is the advantage of dual simplex method ?

(iv) What is the main purpose of sensitivity analysis ?

(v) Define mixed integer programming.

(vi) Define Saddle point.

(vii) When an optimisation problem is called non-linear programming problem (NLPP) ?

(viii) Define convex programming with separable convex objectives.

(ix) Write down general quadratic programming problem in matrix notation.

(x) What is dynamic programming ?

Section–B

2. Show that the following LPP has an unbounded solution :

Max :

$$Z = x_1 + 2x_2$$

s.t. :

$$x_1 - x_2 \leq 4$$

$$x_1 - 5x_2 \leq 8$$

and $$x_1, x_2 \geq 0$$

Or

Give the dual problem of the following LPP :

Minimize :

$$Z = 2x_1 + 2x_2 + 4x_3$$

s.t. :

$$2x_1 + 3x_2 + 5x_3 \geq 2$$

$$3x_1 + x_2 + 7x_3 \leq 3$$

$$x_1 + 4x_2 + 6x_3 \leq 5$$

and $$x_1, x_2, x_3 \geq 0$$

3. Solve the following LPP by using dual Simplex method :

Min. :

$$Z = 5x_1 + 6x_2$$

s.t. :

$$x_1 + x_2 \geq 2$$

$$4x_1 + x_2 \geq 4$$

and $$x_1, x_2 \geq 0$$

Or

The following table gives the optimal solution of LPP :

Max. :

$$Z = -x_1 + 2x_2 - x_3$$

s.t. :

$$3x_1 + x_2 - x_3 \leq 10$$

$$-x_1 + 4x_2 + x_3 \geq 6$$

$$x_2 + x_3 \leq 4$$

and $$x_1, x_2, x_3 \geq 0$$

			C_j	-1	2	-1	0	0	0	$-M$
C_B	B	X_B	b	y_1	y_2	y_3	y_4	y_5	y_6	y_7
0	α_4	x_4	6	3	0	–2	1	0	–1	0
2	α_2	x_2	4	0	1	1	0	0	1	0
0	α_5	x_5	10	1	0	3	0	1	4	–1
			$Z_j - C_j$	1	0	3	0	0	2	M

The optimal solution is :

$x_1 = 0;\ x_2 = 4;\ x_3 = 0$ and Max. Z = 8

Determine the ranges for discrete changes in the components b_2 and b_3 of the requirement vector so as to maintain the optimality of the current optimal solution.

4. Find the integer solution to the LPP :

Max. :

$$Z = x_1 + x_2$$

s.t. :

$$3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

and $$x_1, x_2 \geq 0,$$

x_1, x_2 are integers.

Or

Using bounded variable method, solve the following LPP :

Max. :

$$Z = 3x_1 + x_2 + x_3 + 7x_4$$

s.t. :

$$2x_1 + 3x_2 - x_3 + 4x_4 \leq 40$$

$$-2x_1 + 2x_2 + 5x_3 - x_4 \leq 35$$

$$x_1 + x_2 - 2x_3 + 3x_4 \leq 100$$

and $$x_1 \geq 2,\ x_2 \geq 1,\ x_3 \geq 3,\ x_4 \geq 4$$

5. Write the Kuhn-Tucker conditions for the Non-Linear Programming Problem (NLPP) :

Min. :

$$Z = -2x_1 - x_2 + x_1^2$$

s.t. :

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4$$

and $$x_1, x_2 \geq 0$$

to have optimal solution of at X = X*.

Or

Solve the following by K-T method :

Max. :

$$f(x) = 4x_1 + 2x_2 - x_1^2 - x_2^2 - 5$$

s.t. :

$$x_1 + x_2 \leq 4$$

and $$x_1, x_2 \geq 0$$

6. Write :

$$3x_1^2 + 8x_1x_2 + 16x_2^2 - 8x_3^2$$

in the quadratic form X′AX.

Or

Use dynamic programming to find maximum value of the product $y_1\ y_2\ y_3$:

subject to : $$y_1 + y_2 + y_3 \leq 15$$

and $$y_1, y_2, y_3 \geq 0$$

Section–C

7. Solve the following LPP by using Simplex method :

Minimize :

$$Z = 2x_1 + 9x_2 + x_3$$

s.t. :

$$x_1 + 4x_2 + 2x_3 \geq 5$$

$$3x_1 + x_2 + 2x_3 \geq 4$$

and $$x_1, x_2, x_3 \geq 0$$

8. Use revised Simplex method to solve the following LPP :

Max. :

$$Z = 6x_1 - 2x_2 + 3x_3$$

s.t. :

$$2x_1 - x_2 + 2x_3 \leq 2$$

$$x_1 + 4x_3 \leq 4$$

and $$x_1, x_2, x_3 \geq 0$$

9. Using the bounded variable technique solve the following LPP :

Max. :

$$Z = 3x_1 + 2x_2$$

s.t. :

$$x_1 - 3x_2 \leq 3$$

$$x_1 - 2x_2 \leq 4$$

$$2x_1 + x_2 \leq 20$$

$$x_1 + 3x_2 \leq 30$$

$$-x_1 + x_2 \leq 6$$

and $$0 \leq x_1 \leq 8; \quad 0 \leq x_2 \leq 6$$

10. Use separable programming algorithm to find an approximate optimal solution of NLPP :

Max. :

$$Z = 3x_1 + 2x_2$$

s.t. :

$$4x_1^2 + x_2^2 \leq 16$$

and $$x_1 \geq 0, \ x_2 \geq 0$$

11. Solve the following QPP (Quadratic Programming Problem) by using Wolfe's method :

Max. :

$$Z = 2x_1 + x_2 - x_1^2$$

s.t. :

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4$$

and $$x_1 \geq 0,\ x_2 \geq 0$$