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Total No. of Questions : **11** ] [ Total No. of Printed Pages : **4**

# APF–2168

## M.A./M.Sc. (Final) Examination, 2022

## MATHEMATICS

Paper - Opt-I

## (Generalized Hypergeometric Functions)

*Time : **3** Hours* ] [ *Maximum Marks : **100***

**Section–A** **(Marks : 2 × 10 = 20)**

***Note*** :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

**Section–B** **(Marks : 4 × 5 = 20)**

***Note*** :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

**Section–C** **(Marks : 20 × 3 = 60)**

***Note*** :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

**Section–A**

1. (i) Write Whipple's theorems for the series ${}_3F_2$.

   (ii) Write the convergence conditions for ${}_pF_q$.

   (iii) Define contour integral repesentation for ${}_pF_q$.

(iv) Define Preece and Bailey.

(v) Write the transformation formula for G-function.

(vi) Define Laplace transformation of G-function.

(vii) Write the multiplication formulas for H-function.

(viii) Write any *two* differentiation formulas for the H-function.

(ix) Write finite integrals involving H-functions.

(x) Define contiguous function for H-function.

**Section–B**

2. State and prove Saalschutz' theorem for the series ${}_3F_2$ with unit argument.

***Or***

State and prove the Generalized Hypergeometric Differential equation.

3. If $a$, $b$; so restricted that each of the functions involved exist, then prove that :

$${}_1F_1\left[\begin{matrix} a \\ b \end{matrix}; x\right] \cdot {}_1F_1\left[\begin{matrix} a \\ b \end{matrix}; -x\right] = {}_2F_3\left[\begin{matrix} a, b-a \\ b, \frac{b}{2}, \frac{b}{2}+\frac{1}{2} \end{matrix}; \frac{x^2}{4}\right]$$

***Or***

Prove that :

$$Z^{\sigma} G_{p,q}^{m,n}\left[Z \left| \begin{matrix} \alpha_1, \alpha_2, \ldots \alpha_p \\ \beta_1, \beta_2, \ldots \beta_q \end{matrix} \right.\right] = G_{p,q}^{m,n}\left[Z \left| \begin{matrix} \alpha_1+\sigma, \alpha_2+\sigma, \ldots \alpha_p+\sigma \\ \beta_1+\sigma, \beta_2+\sigma, \ldots \beta_q+\sigma \end{matrix} \right.\right]$$

4. Prove that :

$$G_{2,2}^{1,2}\left[Z \left| \begin{matrix} \frac{5}{2}, & -1 \\ 0, & \frac{5}{2} \end{matrix} \right.\right] = (1+z)^{-2}$$

***Or***

Prove that :

$$\int_0^{\infty} \int x^{\sigma-1} e^{-wx} G_{p,q}^{m,n}\left(\eta x^k\right) dx = \frac{1}{2\pi i} \int_c \phi(s) \eta^s \left\{\int_0^{\infty} e^{-wx} x^{\sigma-ks-1} dx\right\} ds$$

5. Prove that :

$$d(a_p - 1, b_q) \, H(a_1 - 1) - d(b_q, a_1 - 1) \, H(a_p - 1) = -d(a_1 - 1, a_p - 1) \, H(b_q + 1)$$

***Or***

Prove that :

$$\frac{d^r}{dx^r} H_{p,a}^{m,n}\left[\frac{1}{(cx+d)^h}\left|\begin{matrix}(a_p, A_q)\\(b_q, B_q)\end{matrix}\right.\right] = \frac{c^r}{(cx+d)^r} H_{p+1,\,q+1}^{m+1,\,n}\left[(cx+d)^{-h}\left|\begin{matrix}(a_p,\,A_p),\ (1-r,\,h)\\(1,h),\ (b_q,\,B_q)\end{matrix}\right.\right]$$

6. Prove that :

$$(a_1 - a_2) H_{p,q}^{m,n}\left[x\left|\begin{matrix}(a_1,\,A_1)(a_2,\,A_2),\ \ldots\ (a_p,\,A_p)\\(b_1,\,B_1)(b_2,\,B_2),\ \ldots\ (b_q,\,B_q)\end{matrix}\right.\right]$$

$$= H_{p,q}^{m,n}\left[x\left|\begin{matrix}(a_1,\,A_1),(a_2-1,\,A_1),\ (a_3,\,A_3),\ \ldots\ (a_p,\,A_p)\\(b_1,\,B_1),\ \ldots\ \qquad (b_q,\,B_q)\end{matrix}\right.\right]$$

$$- H_{p,q}^{m,n}\left[x\left|\begin{matrix}(a_1-1,\,A_1),\ (a_2,\,A_1),\ (a_3,\,A_3),\ \ldots\ (a_p,\,A_p)\\(b_1,\,B_1),\ \ldots\ \qquad (b_q,\,B_q)\end{matrix}\right.\right]$$

where $n \geq 2$.

***Or***

Prove that :

$$d(a_{p-1} - 1;\ a_{p-2} - 1) \, H(a_p - 1) + d(a_{p-2} - 1,\ a_p - 1) = -d(a_p - 1,\ a_{p-1} - 1) \, H(a_{p-2} - 1)$$

**Section–C**

7. If $n$ is a non-negative integer and if $a$, $b$ are independent of $n$, then prove that :

$${}_3F_2\left[\begin{matrix}-n, a+n,\ \frac{1}{2}+\frac{a}{2}-b\\ 1+a-b,\ \frac{a}{2}+\frac{1}{2}\end{matrix};\ 1\right] = \frac{(b)_n}{(1+a-b)_n}$$

8. If Re $(z) < 0$ and if no $a_m$ or $b_j$ is zero or a negative integer :

$$\frac{1}{2\pi i}\int_{B}\frac{(-z)^{j}\overline{|(-s)}\,\overset{p}{\pi}\overline{|(a_m+s)}}{\overset{q}{\pi}\overline{|(b_j+s)}}=\frac{\overset{p}{\pi}\overline{|a_m}}{\overset{q}{\pi}\overline{|b_j}}\;{}_pF_q\left[\begin{matrix}a_1,\ a_2,\ \ldots\ a_p\\ b_1,\ b_2,\ \ldots\ b_q\end{matrix};z\right]$$

in which B is Barner path of integration.

9. (a) Prove that :

$$z^k\frac{d^k}{dz^k}\left[G_{p,q}^{m,n}\left(z^{-1}\middle|\begin{matrix}a_p\\ b_q\end{matrix}\right)\right]=(-1)^k G_{p+1,q+1}^{m,n+1}\left(z^{-1}\middle|\begin{matrix}1-k,\ a_p\\ b_q,\ 1\end{matrix}\right)$$

(b) Prove that :

$$\int_0^1 y^{-\alpha}(1-y)^{\alpha-\beta-1}G_{p,q}^{m,n}\left[xy\middle|\begin{matrix}a_p\\ b_q\end{matrix}\right]dy=\overline{|(\alpha-\beta)}\,G_{p+1,q+1}^{m,n+1}\left[x\middle|\begin{matrix}\alpha,\ a_p\\ b_q,\ \beta\end{matrix}\right]$$

10. Prove that :

$$\frac{d^r}{dz^r}\left\{z^{-\left(\frac{\nu b_1}{B_1}\right)}H_{p,q}^{m,n}\left[z^{\nu}\middle|\begin{matrix}(a_p,A_q)\\ (b_q,B_q)\end{matrix}\right]\right\}$$

$$=\left(\frac{-\nu}{B_1}\right)^{r}z^{-r-\frac{\nu b_1}{B_1}}H_{p,q}^{m,n}\left[z^{\nu}\middle|\begin{matrix}(a_p,b_q)\\ (r+b_1,B_1),(b_2,B_2),\ \ldots\ (b_q,B_q)\end{matrix}\right]$$

11. Evaluate the following Integral :

$$\int_{-\pi/2}^{\pi/2}(\cos\theta)^{k+l-2}\,e^{i(k-l)\theta}H_{p,q}^{m,n}\left[z(\sec\theta)^{2h}\middle|\begin{matrix}(a_p,\ A_p)\\ (b_q,\ B_q)\end{matrix}\right].d\theta$$