Roll No. : .....

Total No. of Questions : **11**] [Total No. of Printed Pages : **3**

APF–2167

M.A./M.Sc. (Final) Examination, 2022

MATHEMATICS

Paper – VII

(Continuum Mechanics)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) Define primed and coprimed axes.

   (ii) Define $A_i n_i$.

   (iii) Define Permutation symbol.

(iv) What is the use of Gauss divergence theorem in continuum ?

(v) Why the Rarefied gas problem is example for where the continuum approach faits ?

(vi) Define stress vector σ_i.

(vii) Write the internal energy term for first law of Thermodynamics.

(viii) Write the generalized Hooke's law.

(ix) Define homogeneous material in continuum.

(x) Define Strain Energy.

Section–B

2. Prove that :

$$\vec{\nabla} \times \left(\vec{\nabla} A \right) = 0$$

Or

$$\vec{\nabla} r = \frac{\vec{r}}{r}$$

3. Prove that $\in_{ijk} \in_{ist} \sigma_{js} \sigma_{kt}$ is a stress invarient.

Or

If b_{ij} are constant and $b_{ij} = b_{ji}$, then show that :

$$(b_{ij}\, y_i\, y_j)_{,k} = 2\ b_{kj}\, y_j$$

4. Explain the equation of motion in Lagrangian description.

Or

Explain with example about isotropic material in elasticity study.

5. Explain the moment integral theorem for forces.

Or

Prove that $\tilde{u} = \frac{1}{2}\sigma_{ij}\, l_{ij}$ for strain energy.

6. Explain the principle of superposition for elastic problem.

Or

Prove the energy term in first law of Thermodynamics :

$$\frac{du}{dt} = \frac{1}{\rho}\sigma_{ij}\frac{d}{dt}\left(e_{ij}\right) - \frac{1}{\rho}b_{i,j} + c$$

Section–C

7. State and prove Green theorem for scalar point function.

8. Find the force and moment equation of equilibrium for stress in continuum.

9. State and prove second law of thermodynamics.

10. Find the generalized Hooke's law for an isotropic, homogeneous, elastic material as :

$$\sigma_{ij} = \lambda \tilde{D}\delta_{ij} + 2\mu\, l_{ij}$$

11. If $f_i = \in_{ijk} A_j\ B_k$ and $A_i = \in_{ijk} C_{k,j}$, where B_i, C_i are field functions and C_i has continuous derivatives in the region R, then show that :

$$f_i = C_{i,k}\, B_k - C_{k,i}\, B_k$$