Roll No. : ....

Total No. of Questions : **16**] [Total No. of Printed Pages : **4**

PHYSSEM–113

M.Sc. (Ist Semester) Examination Dec., 2022

PHYSICS

Paper - CC–101

(Mathematical Physics)

*Time : **3** Hours*] [*Maximum Marks : **40***

The question paper contains three Sections.

Section–A **(Marks : 1 × 10 = 10)**

Note :– The candidate is required to answer all the *ten* questions carries **1** mark each. The answer should not exceed **50** words.

Section–B **(Marks : 3 × 5 = 15)**

Note :– The candidate is required to answer *five* questions by selecting at least *one* question from each Unit. Each question carries **3** marks. Answer should not exceed **200** words.

Section–C **(Marks : 5 × 3 = 15)**

Note :– The candidate is required to answer *three* questions by selecting at least *one* question from each Unit. Each question carries **5** marks. The answer should not exceed **500** words.

Section–A

1. (i) Define orthogonal and unitary matrices and tell the physical significance of orthogonality.

   (ii) What do you understand by linear transformations of an operator ?

   (iii) What is a complex number ? Tell the physical significance of the real and imaginary parts of a complex number with a proper example.

   (iv) What do you understand by singularities ? How can it be removed from a function ?

   (v) What is Residue ?

   (vi) State the Hermite polynomial differential equation in its standard form.

   (vii) Mention the Rodrigue Formula.

   (viii) What is Laplace transformation and how it differs from Fourier one ?

   (ix) What do you mean by half Fame Expansions ?

   (x) Define Delta function and suggest its importance in the Physics.

Section–B

Unit–I

2. Prove that :

$$(\vec{A} \times \vec{B}) \times \vec{C} = \vec{A} \times (\vec{B} \times \vec{C})$$

when and only when :

$$(\vec{C} \times \vec{A}) \times \vec{B} = 0$$

3. Prove that the eigen vectors of corresponding to distinct eigen values of a matrix are linearly independent.

4. Find the residue of $\frac{z}{(z-a)(z-b)}$ at infinity.

Unit–II

5. Prove that :

$$(2n + 1)xP_n(x) = (n + 1)P_{n-1}(x) + nP_n(x).$$

6. Prove that :

$$\int_{-\infty}^{\infty} e^{x^2} \mathrm{H}m^{(x)}\, \mathrm{H}\,n^{(x)} dx = 0$$

when $m \neq n$.

7. State and find the solution of Lawrence differential equal.

Unit–III

8. Find the Laplace transform :

(a) $t^2 e^{-\mathrm{at}}$

(b) $t^2 \sin \mathrm{at}$

9. Determine the Laplace transform of the half-wave rectified sine wave with period 2π and an amplitude of 2.

10. Find the Fourier transforms of :

$$\mathrm{F}(t) = \begin{cases} 1 - t^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$

Section–C

Unit–I

11. If A is the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{pmatrix}$ and I is the unit matrix of order 3, show that $\mathrm{A}^3 = p\mathrm{I} + q\mathrm{A} + r\mathrm{A}^2$.

12. State and prove the Cauchy's residue theorem.

Unit–II

13. Find power series solution of Hermite equation.

14. Find the singular points of the following differential equation :

(i) $$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2xy = 0$$

(ii) $$x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + xy = 0$$

Unit–III

15. A beam which is clamped at its end $x = 0$ and $x = 1$ carries a uniform load w per unit length. Show that deflection at any point is :

$$y(x) = \frac{wx^2(tx)^2}{24\text{EI}}$$

where E is Young's modulus of elasticity for the beam and I is the moment of Inertia of a cross section of the beam about the axis.

16. Using finite cosine transform solve the equation :

$$\frac{\partial v}{\partial t} = \text{K}\frac{\partial^2 v}{\partial x^2},\ 0 < x < \pi,\ t > 0$$

with boundary conditions :

(i) $\frac{\partial v}{\partial x} = 0$ at $x = 0$ and $x = \pi$, $t > 0$

(ii) V = F(x) at $t = 0$, $0 < x < \pi$