Roll No. : ....

Total No. of Questions : **16**] [Total No. of Printed Pages : **4**

MATHSEM–120

M.A./M.Sc. (Ist Semester) Examination Dec., 2022

MATHEMATICS

Paper – IV

(Calculus of Variation and Special Functions–I)

*Time : **3** Hours*] [*Maximum Marks : **50***

The question paper contains three Sections.

Section–A **(Marks : 1 × 9 = 9)**

Note :– The candidate is required to answer all the *nine* questions carries **1** mark each. The answer should not exceed **50** words.

Section–B **(Marks : 4 × 5 = 20)**

Note :– The candidate is required to answer *five* questions by selecting at least *one* question from each Unit. Each question carries **4** marks. Answer should not exceed **200** words.

Section–C **(Marks : 7 × 3 = 21)**

Note :– The candidate is required to answer *three* questions by selecting at least *one* question from each Unit. Each question carries **7** marks. The answer should not exceed **500** words.

Section–A

1. (i) Explain any one special case of Euler-Lagrange equation.

   (ii) What do you mean by Geodesics ?

   (iii) Write necessary condition for the functional :

$$I = \int_{x_1}^{x_2} f(x, y, y', y'')dx$$

   to be an extremum.

   (iv) Write the Euler-Lagrange equation for the functional :

$$I = [z(x, y)] = \iint \left\{ \left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \right\} dx\, dy$$

   (v) What is transversality condition ?

   (vi) State Hamilton's principle.

   (vii) Prove that :

$${}_2F_1(-n,\ 1;\ 1;\ -x) = (1 + x)^n$$

   (viii) Prove that :

$$\lim_{a \to \infty} \frac{(a)_r}{a^r} = 1$$

   (ix) Write orthogonal properties of Legendre's polynomials.

Section–B

Unit–I

2. Show that the shortest arc length joining the two points is a straight line.

3. Find the external of the functional :

$$I = \int_0^1 (1 + y''^2)dx$$

   subject to the conditions :

$$y(0) = 0,\ y'(0) = 1,\ y(1) = 1,\ y'(1) = 1$$

4. Find any alternative form of Euler-Lagrange equation.

Unit–II

5. Prove that for a functional of the form :

$$I[y(x)] = \int_{x_0}^{x_1} h(x, y)\sqrt{1 + y^{12}}\; dx$$

where $h(x, y) \neq 0$ on the boundary curves, the extremals are orthogonal to the boundary curves.

6. Find the extremal of the functional :

$$\int_{t_1}^{t_2} (\dot{x}^2 + \dot{y}^2)^{1/2} dt$$

where $\dot{x} = \frac{dx}{dt}, \; \dot{y} = \frac{dy}{dt}$.

7. Prove that :

$$\frac{dH}{dt} = \frac{\partial H}{\partial t}$$

where H is the Hamilton's function.

Unit–III

8. Prove that :

$$\int_0^{\pi/2} \frac{d\theta}{(1 - k^2 \sin^2 \theta)} = \frac{\pi}{2} \;\; {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; k^2\right), \;\; |k| < 1$$

9. Prove that :

$$(a - b)_2F_1 = a_2F_1(a+) - b_2F_1(b+)$$

10. Prove that :

$$nP_n = xP_n' - xP_{n-1}'$$

Section–C

Unit–I

11. Show that the time $t[y(x)]$ spent by a particle on translation along a curve of $y = y(x)$, moving with velocity $\frac{ds}{dt} = x$ from the point (0, 0) to the point (1, 1) is minimum if the curve is a circle having its centre on y-axis.

12. Find the extremals of the functionals :

$$I[y(x),\ z(x)] = \int_0^{\pi/2} [2yz + (y')^2 + (z')^2]dx$$

satisfying $y(0) = 0$, $y(\pi/2) = -1$, $z(0) = 0$, $z(\pi/2) = 1$.

Unit–II

13. Find the extremals of the functional :

$$A = \int_{t_1}^{t_2} \frac{1}{2}(x\dot{y} - y\dot{x})dt$$

subject to the integral constraint $\int_{t_1}^{t_2} \sqrt{(\dot{x}^2 + \dot{y}^2)}dt = L$.

14. Find the shortest distance between the circle $x^2 + y^2 = 1$ and the straight line $x + y = 4$.

Unit–III

15. If m is a positive integer, then show that :

$$_2F_1(-m, a+m; c; x) = \frac{x^{1-c}(1-x)^{c-a}}{\left|\overline{(c+m)}\right.} \left|\overline{(c)}\right. \frac{d^m}{dx^m}\left[x^{c+m-1}(1-x)^{a-c+m}\right]$$

16. Establish :

$$\frac{1}{y-x} = \sum_{n=0}^{\infty} (2n+1)P_n(x)Q_n(y)$$