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Total No. of Questions : 16 ]

[ Total No. of Printed Pages : 4

# MATHSEM-119

M.A./M.Sc. (Ist Semester) Examination Dec., 2022

## MATHEMATICS

Paper - III

(Tensor Analysis)

Time : 3 Hours ]

[ Maximum Marks : 50

The question paper contains three Sections.

### Section-A

(Marks :  $1 \times 9 = 9$ )

*Note* :- Answer all the *nine* questions carries 1 mark each. The answer should not exceed 50 words.

### Section-B

(Marks :  $4 \times 5 = 20$ )

*Note* :- Answer *five* questions by selecting at least *one* question from each Unit. Each question carries 4 marks. Answer should not exceed 200 words.

### Section-C

(Marks :  $7 \times 3 = 21$ )

*Note* :- Answer *three* questions by selecting at least *one* question from each Unit. Each question carries 7 marks. The answer should not exceed 500 words.

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### Section–A

1. (i) Define conjugate or reciprocal symmetric tensor.
- (ii) If  $A^{ij}$  is Skew-symmetric and  $B_{ij}$  is symmetric, prove that  $A^{ij}B_{ij} = 0$ .
- (iii) Find the condition of orthogonality of two vectors  $A^i$  and  $B^i$ .
- (iv) Evaluate :

$$[i j, k] + [k j, i]$$

- (v) Obtain the necessary and sufficient condition for a vector  $B^i$ , if variable magnitude to suffer a parallel displacement along a curve C is that

$$B_{ij}^i \frac{dx^j}{ds}.$$

- (vi) Define Geodesics.
- (vii) What is the condition for flat space ?
- (viii) Find the value of :

$$R_{ijk}^\beta + R_{jki}^\beta + R_{kij}^\beta$$

- (ix) Define Covariant curvature tensor.

### Section–B

#### Unit–I

2. Prove that Kronecker delta is a mixed tensor of rank two and it is invariant.
3. Prove that outer multiplication of tensor is commutative and associative.
4. Prove that  $(1, 0, 0, 0)$  and  $(\sqrt{2}, 0, 0, \sqrt{3}/c)$  are unit vectors in  $V_4$  with the metric :

$$ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$$

Also prove that the angle between these vectors is not real.

### Unit-II

5. If  $g_{ij}$  and  $a_{ij}$  are components of two symmetric covariant tensors and  $\left\{ \begin{matrix} i \\ jk \end{matrix} \right\}_g$ ,  $\left\{ \begin{matrix} i \\ jk \end{matrix} \right\}_a$  are the corresponding Christoffel symbols of the second kind then prove that the quantities :

$$\left\{ \begin{matrix} i \\ jk \end{matrix} \right\}_g - \left\{ \begin{matrix} i \\ jk \end{matrix} \right\}_a$$

are components of a mixed tensor.

6. Show that the pole  $P_0$  of the geodesic coordinate system :

$$A_{i,jk} = \frac{\partial^2 A_i}{\partial x^j \partial x^k} - A_l \frac{\partial}{\partial x^k} \left( \begin{matrix} l \\ ij \end{matrix} \right)$$

7. Show that the covariant derivative of an invariant is the same as its ordinary derivative.

### Unit-III

8. For a  $V_2$  referred to an orthogonal system of parametric curve, show that :
- (i)  $R_{12} = 0$
- (ii)  $R_{11}g_{22} = R_{22}g_{11} = R_{1221}$
9. Define Ricci Tensor and explain its properties.
10. Show that a geodesic is an auto parallel curve.

### Section-C

#### Unit-I

11. Prove that the fundamental tensor  $g_{ij}$  is a covariant symmetric tensor of rank two.
12. Prove that :
- (i)  $g^{ij}g^{kl}dg_{ik} = -dg^{jl}$
- (ii)  $g_{ij}g_{kl}dg^{ik} = -dg_{jl}$

### Unit-II

13. The covariant derivative of a covariant vector is symmetric if and only if the vector is gradient.
14. Show that the covariant derivatives of the tensors  $g_{ij}$ ,  $g^{ij}$  and  $\delta$  all vanish identically.

### Unit-III

15. State and prove Bianchi identity.
16. If the metric of a two dimensional flat space is :

$$ds^2 = f(r)[(dx^1)^2 + (dx^2)^2],$$

show that  $f(r) = c(r)^k$ , where  $r^2 = (x^1)^2 + (x^2)^2$  and  $c, k$  are constants.