Roll No. : .....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

BPMS–529

M.Sc. (Previous) Examination, 2023

PHYSICS

Paper - I

(Mathematical Physics and Classical Mechanics)

*Time : **3** Hours*] [*Maximum Marks : **75***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 5 × 5 = 25)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **5** marks.

Section–C **(Marks : 10 × 3 = 30)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **10** marks.

Section–A

1. (i) Show that :

$$A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

is a unitary matrix.

(ii) For any square matrix A, prove that matrices $(A + A^+)$ is Hermitian and $(A - A^+)$ is Skew Hermitian.

(iii) Write any *two* Recurrence formula for Hermite polynomial.

(iv) Find the upper and lower bounds for Legendre polynomial of order *n*.

(v) Write *two* shifting properties of Laplace transform.

(vi) State principle of virtual work.

(vii) For what values of the parameter α, the following transformation is canonical ?

$$Q = q \cos \alpha - p \sin \alpha$$

$$P = q \sin \alpha + p \cos \alpha$$

(viii) Write Poisson's first and second theorems.

(ix) Under which condition the Coriolis force is equal to zero and maximum ?

(x) What do you mean by stable and closed orbits ?

Section–B

2. Find the matrices B such that A = BC :

$$A = \begin{bmatrix} 2 & 3 & -2 \\ 4 & -1 & -2 \\ 0 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ -1 & 2 & 0 \end{bmatrix}$$

Or

Find Eigen value and Eigen vector of matrix :

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

3. Deduce Rodrigue's formula of Legendre polynomial.

Or

For Bessel's function, prove that :

$$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$$

4. Find the Laplace transform of :

(i) $t^2\ e^{-at}$

(ii) $t^2 \sin at$

Or

Find the equation of motion for a particle in a central field of force using Lagrangian method.

5. Derive the equation of motion of a compound pendulum using Hamilton's equation of motion.

Or

Show that the transformation :

$$Q = \sqrt{2q}\ e^{a} \cos p$$

$$P = \sqrt{2q}\ e^{-a} \sin p$$

is a canonical transformation.

6. Explain how the problem of 2 bodies moving under the influence of a mutual central force can be reduced to one body problem.

Or

Deduce Kepler's second law of planetary motion and show that total energy of the system is conserved.

Section–C

7. Diagonalize the matrix :

$$\begin{bmatrix} 4/3 & \sqrt{2}/3 \\ \sqrt{2}/3 & 5/3 \end{bmatrix}$$

by determining the appropriate diagonalizing matrix.

8. Derive the following recurrence relations for Laguerre polynomial :

(i) $(2n + 1 - x)\ L_n(x) = (n + 1)L_{n+1}(x) + nL_{n-1}(x)$

(ii) $L'_{n-1}(x) = L'_n(x) + L_{n-1}(x)$

(iii) $xL'_n(x) = xL_n(x) - nL_{n-1}(x)$

9. Obtain Lagrangian equation for a charged particle in electromagnetic field.

10. Discuss the principle of least action.

11. Discuss the problem of scattering of charged particles by a Coulomb field and obtain Rutherford formula for the scattering cross-section.