Roll No. : ....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

APMA–368

M.A./M.Sc. (Previous) Examination, 2023

MATHEMATICS

Paper – V

(Numerical Methods)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) Write down the Newton's formula for finding the inverse of a number A.

(ii) Find out the initial approximate root of the equation :

$$x - \sin x - 1 = 0$$

(iii) Find the quotient and $f(-4)$ when $f(x) = x^4 + 10x^3 + 39x^2 + 76x + 65$ is divided by $x + 4$.

(iv) Explain Gauss elimination method to solve system of algebraic equations.

(v) Define eigen values and eigen vectors of a Matrix.

(vi) What is Curve Fitting ?

(vii) Write down the Euler's formula for solving ordinary differential equation of fisrt order.

(viii) Explain, in brief, Taylor's series method to solve simultaneous differential equations.

(ix) Define Boundary Value Problem with example.

(x) Define Initial Value Problem and give an example.

Section–B

2. Find the cube root of 10 by applying Newton's formula upto two places of decimal.

Or

By using Newton-Raphson method find the root of $x^4 - x - 10 = 0$ which is nearer to $x = 2$, correct to three decimal places.

3. Perform two interations of the Birge-Vieta method to find the root of equation $f(x) = x^3 - 3x^2 + 4x - 5 = 0$ near $x = 2$.

Or

Solve :

$$x_1 + 2x_2 + x_3 = 8$$
$$2x_1 + 3x_2 + 4x_3 = 20$$
$$4x_1 + 3x_2 + 2x_3 = 16$$

by using Gauss-Jordan Method.

4. Find the eigen values and eigen vectors of the matrix A where :

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

Or

Fit a straight line to the following data regarding as the independent variable :

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

5. Given :

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

with at $x = 0,$

$$y = 1$$

Find y approximately for $x = 0.1$ by Euler's method (*Five* steps).

Or

Use Taylor's series method to solve $\frac{dy}{dx} = x + y \ : \ y(1) = 0$ numerically upto $x = 1.2$ with $h = 0.1$.

6. Solve the boundary value problem :

$$\frac{d^2y}{dx^2} - 64y + 10 = 0$$

$$y\,(0) = y\,(1) = 0$$

by the finite difference method. Compute $y\,(0.5)$.

Or

Find the solution of Boundary Value Problem :

$$\frac{d^2y}{dx^2} = y + x$$

$y\,(0) = 0,\ y\,(1) = 0$

With step size $h = 0.5$

Section–C

7. (i) Find double root of the equation $x^3 - x^2 - x + 1 = 0$ taking initial approximation $x_0 = 0.8$ by using Newton-Raphson Method.

   (ii) Solve the equation $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$ by using bisection method. Correct upto two places of decimal.

8. Solve $x^3 - 8x^2 + 17x - 10 = 0$ by Graeffe's method (squaring *three* times).

9. Compute the largest eigen value and corresponding eigen vector of the following matrix :

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

by using power method with initial eigen vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

10. Using fourth-order Runge-Kutta method solve the differential equation :

$$\frac{dy}{dx} = xy, \; y(1) = 2$$

on the interval [1, 1.4] with $h = 0.2$.

11. Solve the Boundary Value Problem :

$$\frac{d^2y}{dx^2} = y$$

where $y(0) = 0,$

$y(1) = 1$

by shooting method.

Assuming $m_1 = 0.5$ and $m_2 = 0.6$ are initial guesses for exact value of $y'(0) = m$.