Roll No. : ....

Total No. of Questions : **11**] [Total No. of Printed Pages : **3**

APMA–364

M.A./M.Sc. (Previous) Examination, 2023

MATHEMATICS

Paper - I

(Advanced Abstract Algebra)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. Define the following :

   (i) p-group

   (ii) Algebraic extension

   (iii) Nilpotent group

   (iv) Galois extension

(v) Eisentein criterion

(vi) Minimal polynomial

(vii) Annihilator

(viii) Adjoint of a linear transformation

(ix) Quadratic form

(x) Write Bessel's inequality for finite dimensional inner product space.

Section–B

2. Deduce the class equation of symmetric group S_3.

Or

If G is a group of order p^2, p is a prime number, then show that G is abelian.

3. Show that a subgroup of a solvable group is solvable.

Or

Show that homomorphic image of a nilpotent group is nilpotent.

4. Show that a linear operator T can be represented by a diagonal matrix D if there exists a basis B of vector space V consisting of eigen vectors of T.

Or

Show that minimal polynomial $m(t)$ of a Matrix (linear operator) A divides every polynomial that has A as a zero.

5. Find the basis $\{f_1, f_2, f_3\}$ which is dual to the basis $\{e_1, e_2, e_3\}$ of R^3, where $e_1 = (1, -1, 3)$, $e_2 = (0, 1, -1)$, $e_3 = (0, 3, -2)$.

Or

Suppose vector space V is finite dimensional and W is supspace of V. Then show that :

$$\dim W + \dim W^{\circ} = \dim V$$

6. Find the symmetric matrix that corresponds to the quadratic form :

$$q(x, y, z) = 3x^2 + 4xy - y^2 + 8xz - 6yz + z^2$$

Or

Let $u = (\alpha_1, \alpha_2)$, $v = (\beta_1, \beta_2)$. Is $f(u, v) = 2\alpha_1\beta_2 - 3\alpha_2\beta_1$ a bilinear form on R^2 ? Give reason for your answer.

Section–C

7. (i) Determine the simple algebraic extension of the set of rational numbers generated by $2 - x^2$.

   (ii) Show that every finite separable extension is simple.

8. (i) Show that any field of characteristic 0 is perfect.

   (ii) Let K be the field of complex numbers and F is the field of real numbers, then show that K is a normal extension of F.

9. (i) If $f(x)$ and $g(x)$ are primitive polynomials, then show that $f(x)g(x)$ is a primitive polynomial.

   (ii) Show that a scalar λ is eigen value of linear operator T iff linear operator $(\lambda I - T)$ is singular.

10. (i) Let T be a linear operator on a finite dimensional inner product space V(F). Then show that there exists a unique linear operator T* on V such that $(T(u), V) = (u, T^*(v))$, $\forall u_1 v \in V$.

    (ii) Show that the general polynomial of degree $n \geq 5$ is not solvable by radicals.

11. (i) Show that an orthogonal set of non-zero vectors is linearly independent.

    (ii) Find the orthonormal basis for $V_3(R)$ corresponding to the basis (x_1, x_2, x_3), where $x_1 = (1, 1, 1)$, $x_2 = (0, 1, 1)$, $x_3 = (0, 0, 1)$.