Roll No. : .....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

AFMA–269

M.A./M.Sc. (Final) Examination, 2023

MATHEMATICS

Paper – Opt.-IV

(Fluid Dynamics)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) Define equation of continuity.

   (ii) Find the equation of the stream lines for the flow $q = -3y^2 i - 6xj$ at the point (1, 1).

   (iii) Obtain velocity potential and stream function if the complex potential is $\omega = Az^2$.

(iv) What is the difference between sources and sinks ?

(v) Define Shearing stress.

(vi) Explain dynamical similarity.

(vii) Define Prandtl number.

(viii) Write Navier-Stokes equations of motion for a viscous compressible fluid in cartesian coordinates.

(ix) What do you mean by Stokes' first problem ?

(x) Write Stokes' equations for very slow motion.

Section–B

2. Show that :

$$\frac{x^2}{a^2} f(t) + \frac{y^2}{b^2 f(t)} = 1$$

is a possible form of the boundary surface of a liquid.

Or

State and prove Bernoulli's theorem.

3. Use the method of images to prove that if there be a source m at the point z_0 in the fluid bounded by the lines $\theta = 0$ and $\theta = \frac{\pi}{3}$, the solution is :

$$\phi + i\psi = -m \log \left\{ (z^3 - z_0^3)(z^3 - z'^3_0) \right\}$$

where $z_0 = x_0 + iy_0$ and $z'_0 = x_0 - iy_0$.

Or

A cylinder of radius a is placed in a uniform stream of velocity (–U, 0); show that the fluid particles on the surface move according to the law $\tan\left(\frac{\theta}{2}\right) = \tan\left(\frac{\alpha}{2}\right) e^{2Ut/a}$ and never coincide with either stagnation point.

4. The stress tensor at a point p is given by :

$$\sigma_{ij} = \begin{bmatrix} 7 & -5 & 0 \\ -5 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

determine the stress vector on the plane passing through the point p and having for its equation :

$$\frac{x}{4} + \frac{y}{2} + \frac{z}{6} = 1$$

Or

Write short note on viscosity.

5. Discuss rotationally symmetrical flow with stagnation point.

Or

Discuss the flow due to an oscillating plane wall.

6. Write a short note on theory of very slow motions.

Or

Write a short note on Lubrication theory.

Section–C

7. (a) Derive equation of continuity by Lagrangian method.

(b) Derive Helmholtz equations.

8. (a) In irrotational motion in two dimensions, prove that :

$$\left(\frac{\partial q}{\partial x}\right)^2 + \left(\frac{\partial q}{\partial y}\right)^2 = q\nabla^2 q$$

(b) A source and sink of equal strengths are placed equidistant from origin on axis of x. Find complex potential, stream function and velocity at any point in the fluid.

9. Discuss the velocity distribution in the flow of a viscous incompressible fluid between two parallel plates, in the following cases :

   (i) Plane couette flow

   (ii) Plane Poiseuille flow

10. Discuss flow in convergent and divergent channels.

11. Discuss the Stokes' flow past a sphere.