Roll No. : .....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

AFMA–268

M.A./M.Sc. (Final) Examination, 2023

MATHEMATICS

Paper – Opt.-III

(Mechanics)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) Define Moment of Inertia.

   (ii) Define Simple Equivalent Pendulum.

   (iii) Define non-holonomic system.

   (iv) Define Lagrangian function.

(v) Prove that :

$$\frac{d\mathrm{H}}{dt} = \frac{\partial \mathrm{H}}{\partial t}$$

where H is Hamilton's function.

(vi) Define angle variable.

(vii) Define non-homogeneous partial differential equation.

(viii) Find the partial differential equation by the elimination of a and b from :

$$z = ax + by + ab$$

(ix) Write down heat equation in one dimension.

(x) Define mean value formula.

Section–B

2. Find the moment of inertia of uniform solid sphere of radius a mass M about any tangent line.

Or

A ractangular plate swings in a vertical plane about one of its corner. If its period is one second, find the length of the diagonal.

3. A rough uniform rod of length $2a$ is placed on a rough table at right angles to its edge. If its centre of gravity be initially at a distance 'b' beyond the edge, show that the rod will begin to slide when it has turned through an angle $\tan^{-1}\frac{\mu a^2}{a^2 + 9b^2}$, where μ is the coefficient of friction.

Or

A uniform rod of length $2a$ at rest is struck by a blow at right angles to its length at a distance x from its centre. Find the point about which it will begin to turn.

4. Derive Hamilton's Canonical equations of motion.

Or

Discuss Poisson Brackets.

5. Solve :

$$x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$$

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

Or

Solve :

$$pxy + pq + qy - yz = 0$$

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

6. Discuss heat equation.

Or

Discuss wave equation.

Section–C

7. Find the position of the centre of percussion of a sector of a circle, axis is in the plane of the sector, perpendicular to its symmetrical radius and passing through the centre of the circle.

8. Deduce the principle of conservation of energy from the Lagrange's equations.

9. Derive Euler's equations from Hamilton's Canonical equations.

10. Solve :

(i) $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$

where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$.

(ii) $(D^2 - DD' + 2D' - 1)\, z = x^2y^2$

where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$. 10+10=20

11. (i) Find the general solution of two-dimensional Laplace equation.

(ii) Find solution of wave equation in polar forms. 10+10=20