Roll No. : .....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

AFMA–267

M.A./M.Sc. (Final) Examination, 2023

MATHEMATICS

Paper – Opt. II

(Advanced Discrete Mathematics)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) Define predicates and Quantifiers.

   (ii) Define Monoid with example.

   (iii) Define complemented Lattice with example.

   (iv) Show that a Boolean algebra cannot have three elements.

(v) Show that complete bipartite graph $K_{3,3}$ is not a planar graph.

(vi) Define Adjacency matrix.

(vii) Define level of a vertex.

(viii) Define centre of tree.

(ix) Define finite state machine.

(x) Define Regular expression.

Section–B

2. Define biconditional statement and write its truth table.

Or

Let X be the set of people of different heights. Define binary operation $\oplus$ by :

$$x \oplus y = \text{Taller of } x \text{ and } y$$

then (X, $\oplus$) is a monoid.

3. Let a, b, c be elements in a Lattice (L, $\leq$). Show that if $a \leq b$ then

$$a \vee (b \wedge c) \leq b \wedge (a \vee c)$$

Or

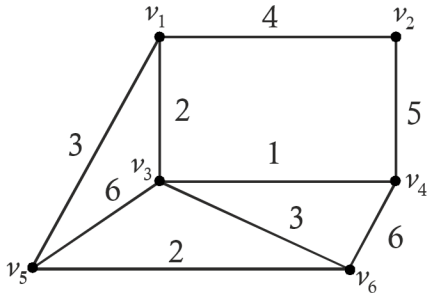
State and prove Bool's theorem.

4. Show that the number of vertices of odd degree is a graph is always even.

Or

Derive Adjacency and Incidence matrix for directed and undirected graph with example.

5. Find the minimal spanning tree for the following graph G using Prim's algorithm :



Or

Show that the number of internal vertices in a binary tree is always less than the number of pendant vertices.

6. Discuss types of Grammar with example.

Or

State and prove pumping Lemma.

Section–C

7. (i) State and prove Law of syllogism.

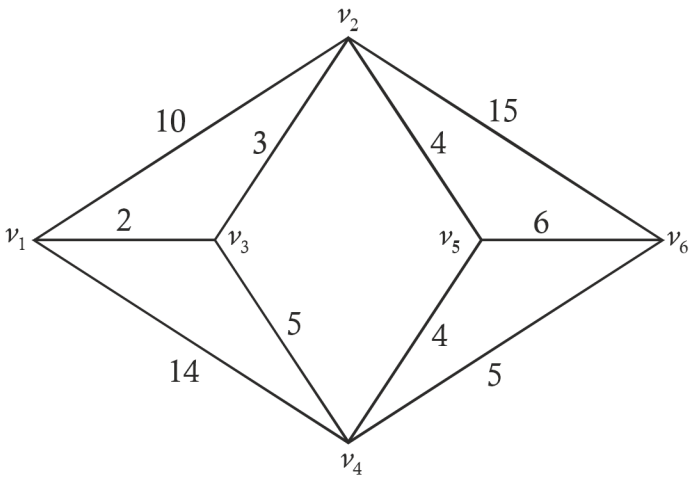
   (ii) Show that if G is a group, then for $a, b \in G$.

$$(ab)^{-1} = b^{-1}a^{-1}$$

8. (i) Show that the dual of a complemented lattice is also a complemented lattice.

   (ii) Show that in any Boolean algebra the order relation '$\leq$' is a partial order Relation.

9. (i) Define weighted graph and find the shortest path from V_1 to V_6 in the following graph using Dijkstra's algorithm.



   (ii) State and prove Kuratowski's theorem.

10. (i) Define the following :

(a) Minimal spanning tree

(b) Kruskal's Algorithm with example.

(ii) Show that if T is binary tree with n vertices and height h then :

$$(h + 1) \leq n \leq (2^{h+1} - 1)$$

11. (i) Show that the language $L\{a^p | p$ is prime$\}$ is not regular.

(ii) State and prove Kleen's theorem.