Roll No. : .....

Total No. of Questions : **11**] [Total No. of Printed Pages : **3**

AFMA–265

M.A./M.Sc. (Final) Examination, 2023

MATHEMATICS

Paper – VII

(Continuum Mechanics)

*Time : **3** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A

1. (i) Write curl of a vector function of position in index notation.

   (ii) Using the index notation, verify the vector identity :

   $$\text{curl grad } A = 0$$

   (iii) Define Normal Stress.

   (iv) Define Stress Deviator Tensor.

(v) Define linear cubical dilation.

(vi) Define Rotation tensor.

(vii) Write the first law of thermodynamics.

(viii) Define rate of cubical dilation.

(ix) Define Irrotational flow.

(x) State principle of superposition.

Section–B

2. Find the components of a vector $A_i \Rightarrow (\sqrt{2}, -\sqrt{2}, 3)$ in a direction normal to the plane $y_1 - y_2 = 0$.

Or

Show that the determinant :

$$|a_{ij}| = \in_{ijk} a_{1i}\, a_{2j}\, a_{3k} = \in_{ijk} a_{i1} a_{j2} a_{k3}$$

where $\in_{ijk}$ is the permutation symbol.

3. In a continuous medium, the components of stress are :

$$\sigma_{ij} = \begin{pmatrix} 2 & 0 & by_2 \\ 0 & 2 & cy_1 \\ by_2 & cy_1 & 2 \end{pmatrix}$$

where b, c are constants. Find the stress vector σ_i at the point $y_i \Rightarrow (1, 2, 3)$ acting on the plane $y_1 + y_2 + y_3 = 6$. Choose the unit normal n_i such that n_i is positive.

Or

Explain body forces and surface forces with examples.

4. Explain principal axis theory for the linear strain tensors.

Or

Prove that the principal strains are all real.

5. Explain the properties of rate of strain tensor.

Or

If $\sigma_{ij} = -p\delta_{ij}$, where p is constant, then show that $du = Tds - pdv$, where $v = \frac{1}{\rho}$ is the specific volume.

6. Prove that elastic constants are components of a Cartesian tensor of the fourth order.

Or

Explain similarity parameters of fluid flow.

Section–C

7. If $F_i \Rightarrow (y_2^2, y_1^2, -y_1 - y_2)$, verify Stokes' theorem by evaluating $\oint_C F_i dy_i$, where C is the boundary of triangle OAB with vertices as O(0, 0, 0), A(1, 0, 0), B(1, 1, 0) and also by evaluating the corresponding surface integral.
8. If $\sigma(1)$, $\sigma(2)$, $\sigma(3)$ are all different, then prove that principal directions are mutually orthogonal.
9. Derive compatibility equations in Lagrangian form for linear strain components.
10. State and prove second law of thermodynamics for reversible and irreversible process.
11. State and prove uniqueness theorem for linear elasticity.