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Total No. of Questions : **11** ] [ Total No. of Printed Pages : **4**

# ASP–645

## M.A./M.Sc. (Final) Examination, 2021

## MATHEMATICS

Paper - Opt.–IV

## (Fluid Dynamics)

*Time : 1½ Hours* ] [ *Maximum Marks : **100***

**Section–A** **(Marks : 2 × 10 = 20)**

***Note*** :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

**(खण्ड–अ)** **(अंक : 2 × 10 = 20)**

**नोट** :– सभी **दस** प्रश्नों के उत्तर दीजिए (उत्तर-सीमा **50** शब्द)। प्रत्येक प्रश्न **2** अंक का है।

**Section–B** **(Marks : 4 × 5 = 20)**

***Note*** :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

**(खण्ड–ब)** **(अंक : 4 × 5 = 20)**

**नोट** :– सभी **पाँच** प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न में विकल्प का चयन कीजिए (उत्तर-सीमा **200** शब्द)। प्रत्येक प्रश्न **4** अंक का है।

**Section–C** **(Marks : 20 × 3 = 60)**

***Note*** :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

**(खण्ड–स)** **(अंक : 20 × 3 = 60)**

**नोट** :– पाँच में से किन्हीं **तीन** प्रश्नों के उत्तर दीजिए (उत्तर-सीमा **500** शब्द)। प्रत्येक प्रश्न **20** अंक का है।

## Section–A

2 each

1. (i) Give two approaches to study the motion of fluid.

   (ii) Define stream lines.

   (iii) State D'Alemberts paradox.

   (iv) Obtain condition of stream function for two dimensional irrotational flow.

   (v) Define source and sink.

   (vi) Give the expression for dilation of the fluid element. Also give its value when fluid is incompressible.

   (vii) Define Mach number.

   (viii) Define Plane Couette flow.

   (ix) Give the boundary condition for flow in convergent and divergent channels.

   (x) Give Stokes' equations for very slow motions.

## Section–B

4 each

2. A mass of fluid is in motion so that the lines of motion lie on the surface of co-axial cylinders. Show that the equation of continuity is :

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial(\rho u)}{\partial \theta}+\frac{\partial(\rho v)}{\partial z}=0$$

$u$ and $v$ are the velocities perpendicular and parallel to $z$-axis.

***Or***

Show that :

$$\frac{x^2}{a^2}f(t)+\frac{y^2}{b^2}\frac{1}{f(t)}=1$$

is a possible form of the boundary surface of a liquid.

3. Show that the curves of constant polential and constant stream function cut orthogonally at their points of intersection.

*Or*

Obtain complex potential for a source situated at origin.

4. State Buckingham $\pi$-theorem.

*Or*

Show that the rate of normal strain at any point P in any direction $\alpha$ is inversely proportional to the square of the length of the radius vector, of the rate of strain quadric at P, drawn in the direction of $\alpha$.

5. Obtain the expression of velocity in case of plane Poiseuille flow.

*Or*

Obtain the expression of velocity in case of flow in tube of circular cross-section.

6. Obtain equations of motion and boundary conditions in case of Stokes' flow past a sphere.

*Or*

Define Lubrication.

**Section–C**

7. (a) Obtain expression for rotational and irrotational motion.

(b) Steam is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and $d$, if V and $v$ be the corresponding velocities of the steam and if the motion be supposed to be that of divergence from the vertex of the cone, prove that $\frac{u}{V}=\frac{D}{d^2}e^{(u^2-v^2)/2k}$.

where $k$ is the pressure divided by the density and supposed constant. 10,10

8. (a) What arrangement of sources and sinks will give rise to the function :

$$w=\log\left(z-\frac{a^2}{z}\right)?$$

Draw a rough sketch of the stream lines. Prove that two of the stream lines sub-divide into the circle $r = a$ and the axis of $y$.

(b) Determine the expression for $\phi$ and $\psi$ for the irrotational motion of an infinite circular cylinder moving in an infine mass of liquid in the direction of $x$-axis with velocity U, the liquid being at rest at infinity. 10,10

9. (a) Discuss Inspection Analysis.

(b) Discuss flow in divergent channel. 10,10

10. (a) Discuss stagnation in two dimensional flow (Hiemenz flow).

(b) Obtain the expression of velocity of unsteady flow is that which sets up from rest when plane wall oscillates with velocity $v_0 \cos nt$ (Stokes' second problem). 10,10

11. Discuss Oseen's flow past a sphere. 20