Roll No : ....

Total No. of Questions : **11**]   [Total No. of Printed Pages : **4**

ASP–643

M.A./M.Sc. (Final) Examination, 2021

MATHEMATICS

Paper - Opt–II

(Advanced Discrete Mathematics)

Time : 1½ Hours]   [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

(खण्ड–अ) **(अंक : 2 × 10 = 20)**

नोट :– सभी **दस** प्रश्नों के उत्तर दीजिए (उत्तर-सीमा **50** शब्द)। प्रत्येक प्रश्न **2** अंक का है।

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

(खण्ड–ब) **(अंक : 4 × 5 = 20)**

नोट :– सभी **पाँच** प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न में विकल्प का चयन कीजिए (उत्तर-सीमा **200** शब्द)। प्रत्येक प्रश्न **4** अंक का है।

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

(खण्ड–स) **(अंक : 20 × 3 = 60)**

नोट :– पाँच में से किन्हीं **तीन** प्रश्नों के उत्तर दीजिए (उत्तर-सीमा **500** शब्द)। प्रत्येक प्रश्न **20** अंक का है।

Section–A

1. (i) Define propositional logic.

   (ii) Define congruence relation.

   (iii) Define complete lattice.

   (iv) Define min terms and max terms.

   (v) Define weighted graph.

   (vi) Define complete bipartite graph.

   (vii) Define minimal spanning tree.

   (viii) Define polish notation.

   (ix) Define finite state machine.

   (x) Define regular grammars.

Section–B

2. Let p, q, r be propositions. Then prove that :

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Or

Let (S, *) and (T, 0) are commutative semigroups. Then show that their product is also commutative semigroup.

3. Let N be the set of positive integers and the relation '$\leq$' be x/y that is "x divides y". Then show that N is a lattice where the join ($\vee$) and meet ($\wedge$) are respectively defined as $a \vee b$ = LCM (a, b) and $a \wedge b$ = HCF (a, b).

Or

For any Boolean algebra B prove that :

$$\forall\ a, b, c \in B (a + b)\cdot(b + c)\cdot(c + a) = a\cdot b + b\cdot c + c\cdot a$$

4. Show that *a* graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty, disjoint subsets v_1 and v_2 such that there exists no edge in G whose end vertex is in subset v_1 and the other in subset v_2.

Or

Explain Warshall's algorithm.

5. Show that there is precisely one path between every pair of vertices in a tree T.

Or

Explain Prim's algorithm.

6. Construct a grammar for the following language :

$$L = \{a^i b^j : i, j \geq 1 \;, \; i \neq j\}$$

Or

Explain Turing Machine.

Section–C

7. (i) Determine the validity of the following argument :

If the market is free then there is no inflation, if there is no inflation then there are price controls. Since there are price controls, therefore the market is free.

(ii) Show that monoids (I, +) and (E, +) are isomorphic, where I is the set of integers and E is the set of even integers.

8. (i) Let (L, ≤) be a distributive lattice and c' be the conjugate of an element c in L. If $b \wedge c' = 0$, then show that $b \leq c$.

(ii) Draw Simplified circuit of the following switch function :

$$F(x, y, z) = x \cdot y \cdot z + x \cdot y' \cdot z + x' \cdot y' \cdot z$$

9. State and prove Euler's formula for connected planar graph.

10. (i) Show that the number of internal vertices in a binary tree is always less than the number of pendant vertices.

(ii) Prove that every connected grapth has at least one spanning tree.

11. (i) Construct a finite automation A which will accept precisely those strings in i_1 and i_2 which have an even number of *i*'s.

(ii) Explain Moore and Mealy machines.