Roll No : ....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

ASP–641

M.A./M.Sc. (Final) Examination, 2021

MATHEMATICS

Paper - VII

(Continuum Mechanics)

Time : 1½ Hours] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions. (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A 2 each

1. (i) Define Kronecker delta.

   (ii) Write the orthogonal component of A_i in the n_i direction.

   (iii) Define curl $\vec{A}$ in index notation.

(iv) Give one example where continuum approach fails.

(v) Define stress tensor and its relation.

(vi) Write the formula for decompose of stress tensor.

(vii) Define Hookean elastic solid.

(viii) Define Youngs modulus of elasticity.

(ix) Write the heat term of first law of thermodynamics.

(x) Write the dissipation function for second law of Thermodynamics.

Section–B 4 each

2. Prove :

$$\vec{\nabla} \times \vec{A} = \in_{ijk} A_{k,j}$$

Or

Prove :

$$\in_{ijk} \in_{kmn} = \delta_{im}\,\delta_{jn} - \delta_{in}\delta_{jn}$$

3. The vector F_i is given by the relationship :

$$F_i = -\phi,\ i$$

where $$\phi = y_1^2 - y_2^2 + y_3^2$$

then show that $f_{i,i} = 0$.

Or

If Q be a point on the stress quadric and if PQ = R, then normal stress at P acting across the surface normal to PQ, is inversely proportional to r^2. Prove it.

4. Explain the equation of motion in Eulerian description.

Or

Explain the isotropic material and homogeneous material in reference to elasticity.

5. Prove for incompressible fluid $\mu = \frac{E}{3}$.

Or

Explain the momentum integral theorem for forces.

6. Explain the energy equation for the Thermodynamics.

Or

Explain the principal of superposition for elastic problems.

Section–C 20 each

7. (a) State and prove first extension of Stokes' theorem.

(b) Show that the determinant :

$$|a_{ij}| = \in_{ijk} a_{1i} a_{2j} a_{3k}$$

where $\in_{ijk}$ is the permutation symbol.

8. (a) Prove that :

$$\sigma_i = \sigma_{ji} n_j$$

(b) Find the principal stress for :

$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & -cy_2 \\ 0 & 0 & cy_1 \\ -cy_2 & cy_1 & 0 \end{bmatrix}$$

where c is a constant.

9. (a) State and prove Reynold transport theorem.

(b) Derive the first law of Thermodynamics.

10. (a) Prove that the absence of body moments, stress tensors are symmetric.

(b) If $A_i(y_1, y_2, y_3)$ is everywhere normal to a closed surface S bounding a region R, then show that :

$$\int_R \in_{ijk} A_{k,j}\, dv = 0$$

11. (a) Prove that elastic constants are tensor of fourth order.

(b) Find the generalized Hook's law in terms of E and ν for isotropic homogeneous linear elastic solid.