Roll No. : ....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

APG–1076

M.A./M.Sc. (Previous) Examination, 2021

MATHEMATICS

Paper - III

(Mathematical Methods)

Time : 1½ Hours] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

Section–A 2 each

1. (i) Find the value of :

$$\frac{d^2}{dx^2}\{{}_2F_1(a,b;c;x)\}$$

(ii) Write the Rodrigues' formula for Legendre polynomial.

(iii) Write the orthogonal property for Bessel function.

(iv) Find the value of $H_3(x)$, where $H_n(x)$ is Hermite polynomial.

(v) Write generating function for Laguerre polynomials.

(vi) Find the inverse Laplace transform :

$$L^{-1}\left\{\log_e\left(\frac{s+1}{s-1}\right)\right\}$$

(vii) Find Laplace transform :

$$L\left\{t\,e^{-at}\sin at\right\}$$

(viii) Define Null Geodesics.

(ix) Show that Kronecker delta is a mixed tensor of rank two and it is invariant.

(x) Show that the number of independent component of Christoffel symbol are :

$$\frac{n^2(n+1)}{2}$$

Section–B 4 each

2. Prove that :

$$2x\,H_n(x) = 2n\,H_{n-1}(x) + H_{n+1}(x)$$

Or

Prove that :

$$\int_0^{\pi/2}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} = \frac{\pi}{2}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2};1;k^2\right)$$

3. Prove that Beltrami's result :

$$(2n+1)(x^2-1)\,P_n' = n(n+1)(P_{n+1}-P_{n-1})$$

Or

Find the value of :

$$I = \int x^3 J_0(x)\, dx$$

4. Show that :

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^r} (\sqrt{g}\, B^{pqr})$$

is a tensor, where B^{pqr} is skew-symmetric tensor.

Or

Express div A^i in term of physical component of cylindrical polar co-ordinates.

5. Prove Bianchi identity :

$$R^{\beta}_{ijk,l} + R^{\beta}_{ikl,j} + R^{\beta}_{ilj,k} = 0$$

Or

For the V_2 whose line element is given :

$$dS^2 = du^2 + G^2\, dv^2$$

where G is function of u and v. Show that :

$$R_{1212} = \ominus G \frac{\partial^2 G}{\partial u^2}$$

6. Find the Laplace transform of $\sin\sqrt{x}$. Hence find :

$$L\left\{\frac{\cos\sqrt{x}}{\sqrt{x}}\right\} = \left(\frac{\pi}{s}\right)^{1/2} e^{-1/4s}$$

Or

Solve by using Laplace transform $y'' + 4y' + 3y = e^{-t}$, $y \equiv y(t)$ given that $y(0) = y'(0) = 1$.

Section–C

20 each

7. (a) Prove that :

$$L_n(x) = \frac{e^x}{\lfloor n} \frac{d^n}{dx^n}(x^n e^{-x})$$

(b) Prove that :

$$H_n(x) = 2^n \left\{ \exp\left(-\frac{1}{4} \frac{d^2}{dx^2} \right) \right\} x^n$$

8. (a) Prove that all roots of $P_n(x) = 0$ are real and lies between –1 and +1.

(b) Prove that :

$$J_n(x) = (-2)^n x^n \frac{d^n}{d(x^2)^n} \{J_0(x)\}$$

9. (a) Calculate the Christoffel symbols corresponding to metric :

$$dS^2 = (dx')^2 + (x')^2 (dx^2)^2 + (x')^2 (\sin x^2)^2 (dx^3)^2$$

(b) The covariant derivative of a covariant vector is symmetric if and only if the vector is gradient.

10. (a) Show that on the surface of a sphere, all the great circles are geodesics while no other circle is geodesics.

(b) If the metric of a two-dimensional flat space is :

$$dS^2 = f(r) [(dx')^2 + (dx^2)^2]$$

Show that $f(r) = c\, r^k$, where $r^2 = (x')^2 + (x^2)^2$ here c, k are constants.

11. (a) State Convolution theorem. Using Convolution theorem find :

$$L^{-1} \left\{ \frac{1}{(S+2)^2 (S-2)} \right\}$$

(b) $ty''(t) + (1 - 2t)\, y'(t) - 2y(t) = 0$ given that $y(0) = 1$ and $y'(0) = 2$.