Roll No. : .....

Total No. of Questions : **11**] [Total No. of Printed Pages : **4**

APG–1078

M.A./M.Sc. (Previous) Examination, 2021

MATHEMATICS

Paper - V

(Numerical Methods)

*Time : **1½** Hours*] [*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions are compulsory. Each question is to be attempted in around **50** words. Each question carries **2** marks.

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice. (Answer limit **200** words). Each question carries **4** marks.

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five. (Answer limit **500** words). Each question carries **20** marks.

Section–A 2 each

1. (i) Write down Newton's formula for finding cube root of a number.

   (ii) Find $(8)^{1/3}$ by Newton's formula upto two decimal points.

   (iii) Define Algebraic equation and roots of an equation.

(iv) Write down the Newton-Raphson formula of a polynomial equation $f(x) = 0$ for multiple roots.

(v) Define characteristic equation and characteristic roots of a square matrix A.

(vi) Find the eigen values of :

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

(vii) Use Euler's modified method with one step to solve :

$$\frac{dy}{dx} = x^2 + y$$

with $y(0) = 0.94$. Find $y(0.1)$.

(viii) State Cayley–Hamilton theorem for square matrix A.

(ix) Define Boundary Value Problem.

(x) For the polynomial $f(x) = 2x^3 - 6x + 13$, find $f(2)$ and $f'(2)$ by using synthetic division method.

Section–B 4 each

2. By using Newton–Raphson method, find the root of equation $x^4 - x - 10 = 0$ which is nearer to $x = 2$, correct to three places of decimal.

Or

Find a real root of equation $x^3 + x^2 - 1 = 0$ by iteration method, correct to two places of decimals.

3. Using Gauss's elimination method, solve the following system of linear equations :

$$2x + y + z = 10$$

$$3x + 2y + 3z = 18$$

$$x + 4y + 9z = 16$$

Or

By Graeffe's root squaring method, compute the roots of the equation $x^3 - 7x^2 + 10x - 2 = 0$ perform 3 squarings.

4. Fit a straight line to the following data regarding x as the independent variable :

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

Or

Use power method to find the largest eigen value and corresponding eigen vector of the matrix A where :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

State with initial vector :

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

5. Use Taylor's series method to find the numerical solution of $\frac{dy}{dx} = 1 + xy,$ when $x = 0$, $y = 2$ at $x = 0.4$. (Take $h = 0.4$)

Or

Use Euler's method to determine an approximate value of y at $x = 0.2$ from the IVP :

$$\frac{dy}{dx} = 1 - x + 4y; \quad y(0) = 1$$

taking the step size $h = 0.1$.

6. Use finite difference method to solve the boundary value problem :

$$\frac{d^2y}{dx^2} = y; \quad y(0) = 0$$

$$y(1) = 1.8$$

with $h = 0.25$.

Or

Solve the boundary value problem $y'' = xy$

$y(0) + y'(0) = 1$ and $y(1) = 1$ with step size $h = 1/3$.

Section–C 20 each

7. Solve $\log x = \cos x$ to five places of decimals by Newton-Raphson method.

8. Solve the following system of equations by Jocobi's iteration method :

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

9. For the data given below, find the equation to the best fitting exponential curve of the form $Y = ae^{bX}$:

X	1	2	3	4	5	6
Y	1.6	4.5	13.8	40.2	125	300

10. Apply Runge-Kutta method to find approximate value of y for $x = 0.2$ in steps of 0.1, if :

$$\frac{dy}{dx} = x + y^2$$

given that $y = 1$ where $x = 0$.

11. Using Shooting Method, solve the boundary value problem $y''(x) = y(x)$, $y(0) = 0$ and $y(1) = 1.17$ by taking the initial guesses for $y'(0) = m$; $m_0 = 0.8$ and $m_1 = 0.9$.